# PERSECUTOR — PREY SYSTEM RESOLUTION FOR THE PERSECUTOR $\mathbb{R}^2$ -DEFINED EXPONENTIAL PROBABILITY OF STRIKING THE PREY WITH ADDED [0; 1]×[0; 1]-SQUARE KNOWN WHITE NOISE

There has been considered the persecutor — prey antagonistic system with the non-ideal exponential kernel on the  $[0;1] \times [0;1]$ -square. The updated program module ppsr2, working with the accuracy parameter  $\alpha$  and taking into account the kernel roughness, has been elaborated. Besides, there has been formed the MATLAB 7.0.1 application eppsr2 for the visualized praxis in working with the investigated simplest antagonistic game persecution model. All the being returned solutions of the antagonistic games, if only they are not exact analytically, may be re-obtained once again in any high precision, that needed.

Рассмотрено антагонистическую систему преследователь — добыча с неидеальным экспоненциальным ядром на [0;1]×[0;1]-квадрате. Разработан обновленный программный модуль ppsr2, работающий с параметром меткости  $\alpha$  и учитывающий шероховатость ядра. Кроме того, в MATLAB 7.0.1 сделано форму приложения eppsr2 для визуализированных упражнений при работе с исследованной простейшей моделью преследования в виде антагонистической игры. Все возвращаемые решения антагонистических игр, если только они не являются аналитически точными, могут быть получены каждый раз снова с как угодно высокой точностью.

## A problem disposition and the referred origins

Conflict events and processes are modeled with various branches of the game theory. In the persecution models there take their solid place the differential games. But the solution process of the differential games is generally harder, than solving the antagonistic game. Hence, the persecution models reduction to the antagonistic game is pretty preferable to building the differential game model and searching its solution. Then may it be considered the following problem. There is the first player, shooting the goal, maneuvering with some overload y. For striking the goal there is applied the special gadget with its overload x, that is founded on a hypothesis about the goal move. Normalizing those overloads, may they be  $x \in [0; 1]$  and  $y \in [0; 1]$ . The task of the first player, for further being called the persecutor, is to strike the opponent, for further being called the prey. And the task of the prey is to stay unstruck. This is the known persecution problem, modeled as the antagonistic game, in which the kernel is the probability

$$P(x, y) = \exp\left[-\alpha (x - y)^2\right]$$
(1)

of striking the prey, defined on the unit square  $D_p = [0, 1] \times [0, 1]$  by some parameter  $\alpha > 0$  [1, p. 62 — 63]. Though there are some definite solutions [1, p. 63 — 66] for this game, given for fixed values of the parameter  $\alpha$ , the problem point until the paper [2] promulgation had laid in resolving that conflict system for any  $\alpha > 0$ . The needful resolution had been found and formed within the MATLAB 7.0.1. However, it would be ideal to have certainly the surface (1) as the kernel of the corresponding antagonistic game. Truly the kernel only reminds the surface (1), and the real kernel is the surface (1) with the  $[0; 1] \times [0; 1]$ -square uniformly distributed rough edges, which may be called the white noise n(x, y). So,

$$P(x, y) = \exp\left[-\alpha(x-y)^{2}\right] + n(x, y)$$
<sup>(2)</sup>

is the kernel of the persecution antagonistic game by taking into account the noised exponential probability (1). The four examples of the surface (2), plotted by the different  $\alpha$  at the fixed noise level, are given on the figures 1 — 4.

#### The paper aim

Due to the said, there are the known parameter  $\alpha$  and the surface (2), describing the nearly exponential probability of striking the prey. This simplest persecution model may reflect some important features of the conflict pair, which are very hard to be detected by the appropriate differential game model. Therefore, the disposed problem paper aim is to solve the antagonistic game with the kernel (2) in the analytic way for as wide as possible the parameter  $\alpha$  range, and the rest of this range must be solved numerically. The antagonistic game resolution should be formed within a program module for the fast visualized representation of the optimal behavior of the persecutor and prey.

### The exponential probability kernel

Now turn backward the paper [2] and resume the obtained there results. Setting n(x, y) = 0, start recalling, that it had been reasonable to check if the stated game with the kernel (1) is either convex or concave. That will help to solve it with the known algorithm for convex-concave antagonistic games [3 - 5]. The first derivative of the function (1) by the variable x is

$$\frac{\partial P(x, y)}{\partial x} = \frac{\partial}{\partial x} \left( \exp\left[ -\alpha \left( x - y \right)^2 \right] \right) = -2\alpha \left( x - y \right) \exp\left[ -\alpha \left( x - y \right)^2 \right]$$
(3)

and its second derivative is

$$\frac{\partial^2 P(x, y)}{\partial x^2} = \frac{\partial}{\partial x} \left( -2\alpha (x - y) \exp\left[ -\alpha (x - y)^2 \right] \right) = -2\alpha \exp\left[ -\alpha (x - y)^2 \right] + 4\alpha^2 (x - y)^2 \exp\left[ -\alpha (x - y)^2 \right] = \\ = 2\alpha \left( 2\alpha (x - y)^2 - 1 \right) \exp\left[ -\alpha (x - y)^2 \right].$$
(4)

The concavity condition  $\frac{\partial^2 P(x, y)}{\partial x^2} \leq 0$  must be true  $\forall x \in [0; 1]$  and  $\forall y \in [0; 1]$ . As there is the triple inequality

$$0 < \exp(-\alpha) \leq \exp\left[-\alpha \left(x - y\right)^2\right] \leq 1,$$
(5)

then the inequality

$$2\alpha \Big(2\alpha (x-y)^2 - 1\Big) \exp\left[-\alpha (x-y)^2\right] \leq 0$$
(6)

is identical to the inequality

$$2\alpha (x-y)^2 - 1 \leqslant 0, \qquad (7)$$



Figure 2. The surface (2) by  $\alpha = 0.2$ 

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Figure 4. The surface (2) by  $\alpha = 0.7$ 

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whence, minding the obvious factor  $(x-y)^2 \in [0;1]$ , the parameter  $\alpha \in \left(0;\frac{1}{2}\right]$  determines the concavity of the antagonistic game with the kernel (1).

Continuing, now clear out whether this game is convex or not. The first derivative of the function (1) by the variable y is

$$\frac{\partial P(x, y)}{\partial y} = \frac{\partial}{\partial y} \left( \exp\left[ -\alpha (x - y)^2 \right] \right) = 2\alpha (x - y) \exp\left[ -\alpha (x - y)^2 \right]$$
(8)

and its second derivative

$$\frac{\partial^2 P(x, y)}{\partial y^2} = \frac{\partial}{\partial y} \left( 2\alpha (x - y) \exp\left[ -\alpha (x - y)^2 \right] \right) = -2\alpha \exp\left[ -\alpha (x - y)^2 \right] + 4\alpha^2 (x - y)^2 \exp\left[ -\alpha (x - y)^2 \right] = 2\alpha \left( 2\alpha (x - y)^2 - 1 \right) \exp\left[ -\alpha (x - y)^2 \right]$$
(9)

turns to be identical to the second derivative (4) of the function (1) by the variable x. Then the corresponding convexity condition

$$2\alpha(x-y)^2 - 1 \ge 0 \tag{10}$$

is impracticable for any  $\alpha > 0$  as this fails yet for x = y.

Accepting the parameter  $\alpha \in \left(0, \frac{1}{2}\right]$  after the stated (3) — (10), the minimum of the surface (1) as the function of the variable *y* on the segment [0, 1] is

$$\min_{y \in [0;1]} P(x, y) = \min_{y \in [0;1]} \exp\left[-\alpha (x - y)^2\right] = P(x, 1) = \exp\left[-\alpha (x - 1)^2\right] \quad \forall \ x \in \left[0; \frac{1}{2}\right]$$
(11)

and

$$\min_{y \in [0;1]} P(x, y) = \min_{y \in [0;1]} \exp\left[-\alpha (x - y)^2\right] = P(x, 0) = \exp\left(-\alpha x^2\right) \quad \forall \ x \in \left[\frac{1}{2}; 1\right].$$
(12)

The optimal game value

$$v_{\text{opt}} = \max_{x \in [0; 1]} \min_{y \in [0; 1]} P(x, y) = \max\left\{\max_{x \in [0; \frac{1}{2}]} P(x, 1), \max_{x \in [\frac{1}{2}; 1]} P(x, 0)\right\} = \\ = \max\left\{\max_{x \in [0; \frac{1}{2}]} \left(\exp\left[-\alpha(x-1)^{2}\right]\right), \max_{x \in [\frac{1}{2}; 1]} \left[\exp\left(-\alpha x^{2}\right)\right]\right\} = \exp\left(-\alpha\left(\frac{1}{2}\right)^{2}\right) = \exp\left(-\frac{\alpha}{4}\right)$$
(13)

is reached on the optimal pure strategy  $x_{opt} = \frac{1}{2}$  of the first player. The essential pure strategies of the second player are the roots of the standard equation

$$v_{\text{opt}} = \exp\left(-\frac{\alpha}{4}\right) = P\left(x_{\text{opt}}, y\right) = \exp\left[-\alpha\left(x_{\text{opt}} - y\right)^2\right] = P\left(\frac{1}{2}, y\right) = \exp\left[-\alpha\left(\frac{1}{2} - y\right)^2\right].$$
 (14)

Going on, as  $-\frac{\alpha}{4} = -\frac{\alpha}{4} + \alpha y - \alpha y^2$  and y(1-y) = 0 then the roots of the equation (14) are the pure strategies  $y = y_1 = 0$  and  $y = y_2 = 1$ .

Further on, may  $\varphi(y)$  be the optimal probability of the pure strategy y selection by the second player. Then the optimal probability of the pure strategy  $y_1 = 0$  selection is determined from the equilibrium situation nonstrict inequality

$$P(x, y_{1})\phi(y_{1}) + P(x, y_{2})\phi(y_{2}) = P(x, y_{1})\phi(y_{1}) + P(x, y_{2})[1-\phi(y_{1})] =$$

$$= P(x, 0)\phi(0) + P(x, 1)[1-\phi(0)] = \phi(0)\exp(-\alpha x^{2}) + [1-\phi(0)]\exp[-\alpha(x-1)^{2}] \leq$$

$$\leq P(x_{opt}, y_{1})\phi(y_{1}) + P(x_{opt}, y_{2})\phi(y_{2}) = P(x_{opt}, y_{1})\phi(y_{1}) + P(x_{opt}, y_{2})[1-\phi(y_{1})] =$$

$$= \phi(0)\exp(-\frac{\alpha}{4}) + [1-\phi(0)]\exp(-\frac{\alpha}{4}) = \exp(-\frac{\alpha}{4}) = v_{opt}$$
(15)

by  $x \neq x_{opt} = \frac{1}{2}$ , where just has been used the equilibrium point conception with the condition  $\varphi(y_1) + \varphi(y_2) = 1$  [6 — 10]. In order of (15) there is the subsequent inequality

$$\varphi(0)\Big(\exp\left(-\alpha x^2\right) - \exp\left[-\alpha \left(x-1\right)^2\right]\Big) \leqslant \exp\left(-\frac{\alpha}{4}\right) - \exp\left[-\alpha \left(x-1\right)^2\right],\tag{16}$$

which components should be evaluated for  $x \in \left[0; \frac{1}{2}\right)$  and  $x \in \left(\frac{1}{2}; 1\right]$  by the corresponding minima (11) and (12) of the kernel (1).

As it is easy to see (figure 5), there is the relation

$$\exp(-\alpha x^{2}) > \exp\left[-\alpha (x-1)^{2}\right]$$
(17)

 $\forall x \in \left[0; \frac{1}{2}\right)$  and that conditionally gives

$$\varphi(0) \leqslant \frac{\exp\left(-\frac{\alpha}{4}\right) - \exp\left[-\alpha(x-1)^{2}\right]}{\exp\left(-\alpha x^{2}\right) - \exp\left[-\alpha(x-1)^{2}\right]}.$$
(18)



Figure 5. Relation between two exponents in the left side of the inequality (16)

The ratio in the right side of the inequality (18) is the monotonously decreasing curve by any fixed  $\alpha \in \left(0; \frac{1}{2}\right)$  and  $\forall x \in \left[0; \frac{1}{2}\right)$  (figure 6). But the limit



Figure 6. The right side of the inequality (18), that imaged as a surface

$$\lim_{\substack{x \to \frac{1}{2} \\ \varepsilon > 0^{2}}} \left( \frac{\exp\left(-\frac{\alpha}{4}\right) - \exp\left[-\alpha(x-1)^{2}\right]}{\exp\left(-\alpha x^{2}\right) - \exp\left[-\alpha(x-1)^{2}\right]} \right) = \lim_{\substack{x \to \frac{1}{2} \\ \varepsilon > 0^{2}}} \left( \frac{\frac{\partial}{\partial x} \left(\exp\left(-\frac{\alpha}{4}\right) - \exp\left[-\alpha(x-1)^{2}\right]\right)}{\frac{\partial}{\partial x} \left(\exp\left(-\alpha x^{2}\right) - \exp\left[-\alpha(x-1)^{2}\right]\right)} \right) = \lim_{\substack{x \to \frac{1}{2} \\ \varepsilon > 0^{2}}} \left( \frac{2\alpha(x-1)\exp\left[-\alpha(x-1)^{2}\right]}{-2\alpha x \exp\left(-\alpha x^{2}\right) + 2\alpha(x-1)\exp\left[-\alpha(x-1)^{2}\right]} \right) = \lim_{\substack{x \to \frac{1}{2} \\ \varepsilon > 0^{2}}} \left( \frac{-\alpha \exp\left(-\frac{\alpha}{4}\right)}{-\alpha \exp\left(-\frac{\alpha}{4}\right) - \alpha \exp\left(-\frac{\alpha}{4}\right)} \right) = \frac{1}{2}$$
(19)

displays that  $\forall x \in \left[0; \frac{1}{2}\right]$  the optimal probability of the pure strategy  $y_1 = 0$  selection satisfies the condition  $\phi(0) \in \left[0; \frac{1}{2}\right] = \Phi_1.$ 

Taking  $x \in \left(\frac{1}{2}; 1\right]$  leads from the statement (16) through the relation

$$\exp\left(-\alpha x^{2}\right) < \exp\left[-\alpha \left(x-1\right)^{2}\right]$$
(21)

to the inequality

$$\varphi(0) \ge \frac{\exp\left(-\frac{\alpha}{4}\right) - \exp\left[-\alpha(x-1)^{2}\right]}{\exp\left(-\alpha x^{2}\right) - \exp\left[-\alpha(x-1)^{2}\right]}.$$
(22)

The ratio in the right side of the inequality (22) is the monotonously decreasing curve by any fixed  $\alpha \in \left(0; \frac{1}{2}\right]$  and  $\forall x \in \left(\frac{1}{2}; 1\right]$  (figure 7). But the limit

(20)

 $\frac{\exp\left(-\frac{\alpha}{4}\right) - \exp\left[-\alpha(x-1)^2\right]}{\exp\left(-\alpha x^2\right) - \exp\left[-\alpha(x-1)^2\right]}$ 

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Figure 7. The right side of the inequality (22), that imaged as a surface

$$\lim_{\substack{x \to \frac{1}{2} + \varepsilon} \varepsilon > 0} \left( \frac{\exp\left(-\frac{\alpha}{4}\right) - \exp\left[-\alpha\left(x-1\right)^{2}\right]}{\exp\left(-\alpha x^{2}\right) - \exp\left[-\alpha\left(x-1\right)^{2}\right]} \right) = \frac{1}{2}$$
(23)

displays that  $\forall x \in \left(\frac{1}{2}; 1\right]$  the optimal probability of the pure strategy  $y_1 = 0$  selection satisfies the condition

$$\varphi(0) \in \left[\frac{1}{2}; 1\right] = \Phi_2 . \tag{24}$$

Consequently, the optimal probability of the pure strategy  $y_1 = 0$  selection is

$$\varphi(0) \in \Phi_1 \cap \Phi_2 = \left[0; \frac{1}{2}\right] \cap \left[\frac{1}{2}; 1\right] = \left\{\frac{1}{2}\right\}.$$

$$(25)$$

Hence the optimal probability of the pure strategy  $y_2 = 1$  selection is  $\varphi(1) = \frac{1}{2}$ . Then the game with the defined on the unit square  $D_p = [0; 1] \times [0; 1]$  kernel (1) by the parameter  $\alpha \in \left(0; \frac{1}{2}\right]$  is solved in the single optimal strategy of the first player  $x_{opt} = \frac{1}{2}$  and the single mixed optimal strategy of the second player, which consists in the equiprobable selection of the two pure strategies  $y_1 = 0$  and  $y_2 = 1$  due to (19) — (25). Such equilibrium situation gives the optimal game value  $v_{opt} = \exp\left(-\frac{\alpha}{4}\right)$ . And by this system persecutor — prey configuration the likelihood of striking the prey is sufficiently great, as even in the worst case, that is by  $\alpha = \frac{1}{2}$ , the probability of the strike is  $v_{opt} = \exp\left(-\frac{1}{8}\right) \approx 0.8825$ .

However, the parameter  $\alpha \in \left(0; \frac{1}{2}\right]$  is a feature of the high-technology equipment for the persecutor. So, the

case  $\alpha > \frac{1}{2}$  should be solved carefully as well as the previous. The exact analytic methods of solving such system persecutor — prey configuration are unavailable. Then for getting the needful solution there may be applied the MATLAB 7.0.1 software for numerical computations. The viewed on the figure 8 program module ppsr (Persecutor — Prey System Resolution) handles the single input  $\alpha$  and returns the result as the solution of the game with the defined on the unit square  $D_P = [0; 1] \times [0; 1]$  kernel (1). For instance, if  $\alpha = \frac{3}{4}$  then the solution is much the same as for the cases with  $\alpha \in \left(0; \frac{1}{2}\right]$  (figure 9). Actually, the carried investigations prompt that for  $\alpha \in (0; 2]$  the solution stays nearly stable. Some insignificant deviation from the equiprobable selection of the two pure strategies  $y_1 = 0$  and  $y_2 = 1$  is explained with the finite precision of calculations, which mainly are in the program submodule sp, accepting the kernel in the matrix form and returning the whole game solution (figure 10). Nevertheless, the single optimal pure strategy of the persecutor  $x_{opt} = \frac{1}{2}$  remains for  $\alpha \in (0; 2]$  definitely. And only for  $\alpha > 2$  the number of pure strategies of the players, selected with nonzero optimal probabilities, starts increasing (figures 11 — 19).

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1	function [PIPS PIPS probabilities P2PS P2PS probabilities v ont1 = nmer(alnha)
2 -	if alpha <= 0
3 -	error/' The input parameter alpha must be greater than 0.')
4 -	end
5 -	if alpha <= 0.5
6 -	disp(' x opt = 1/2')
7 -	disp(' Y opt= $\langle (0, 1), (1/2, 1/2) \rangle$ , that is there the pure strategies v=0 and v=1 are selected equiprobably')
8 -	disp(['vopt = 'num2str(exp(-alpha/4))])
9 -	P1PS = 0.5;
10 -	P1PS probabilities = 1;
11 -	P2PS = [0 1];
12 -	P2PS_probabilities = [0.5 0.5];
13 -	v_opt = num2str(exp(-alpha/4));
14 -	else
15 -	kx = 0;
16 -	N = 200;
17 -	for x = 0:1/N:1
18 -	kx = kx + 1;
19 -	ky = 0;
20 -	for $y = 0:1/N:1$
21 -	ky = ky + 1;
22 -	$P(kx, ky) = exp(-alpha^{*}(x-y)^{2});$
23 -	end
24 -	end
25 -	[A_opt, r_opt, vlow, vup, ons, v_opt] = sp(r);
20 -	Difficuence and the provided of the provided
28 -	$P2PS_{numbers} = find(abs(x_{-pp}) > i = -b),$
29 -	disn(' The persecutor must strategies to be selected.')
30 -	ares (PIPS another and School and
31 -	disp(' The optimal probabilities of the persecutor pure strategies selection:')
32 -	PIPS probabilities = X opt/find(abs/X opt) > $1e-201$ );
33 -	if sum(PIPS probabilities > 1-1e-10) > 0
34 -	P1PS probabilities = 1;
35 -	disp(' The single optimal pure strategy of the persecutor must be selected in every play.')
36 -	else
37 -	P1PS_probabilities = X_opt(find(abs(X_opt) > 1e-20))
38 -	end
39 -	disp('')
40 -	disp(' The prey pure strategies to be selected:')
41 -	P2PS = (P2PSnumbers-1)/N
42 -	disp(' The optimal probabilities of the prey pure strategies selection:')
43 -	P2PS_probabilities = Y_opt(find(abs(Y_opt) > 1e-20))
44 -	disp('')
45 -	disp([' v_opt = ' num2str(v_opt)])
46 -	ena den ciulto
47 -	alsop('')
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Figure 8. The program module ppsr code in MATLAB 7.0.1 M-file Editor



Figure 9. The persecutor — prey system resolution for  $\alpha = 0.75$ 



Figure 10. The program submodule sp code in MATLAB 7.0.1 M-file Editor

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The persecutor pure strategies to be selected:	
P1PS =	
0.0700 0.9300	
The optimal probabilities of the persecutor pure strategies selection:	
P1PS_probabilities =	
0.5000 0.5000	
The prey pure strategies to be selected:	
P2PS =	
0 1	
The optimal probabilities of the prey pure strategies selection:	
P2PS_probabilities =	
0.5000 0.5000	
v_opt = 0.53004	-
4 Start	

Figure 11. Appearance of the two pure strategies  $x_1 = 0.03$  and  $x_1 = 0.97$  for the equiprobable selection of the persecutor by  $\alpha = 3$ 



Figure 12. By  $\,\alpha=4\,$  both the players have the three pure strategies for their selection



Figure 13. Symmetrization of the three pure strategies and their selection probabilities by  $\alpha = 8$ 

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>> [P1PS P1PS_probabilities P2PS P2PS_probabilities v_opt] = ppsr(11);	
The persecutor pure strategies to be selected:	
P1PS =	
0.0550 0.0600 0.5000 0.9400 0.9450	
The optimal probabilities of the persecutor pure strategies selection:	
P1PS_probabilities =	
0.2509 0.1080 0.2822 0.1080 0.2509	
The prey pure strategies to be selected:	
P2PS =	
0 0.4300 0.4350 0.5650 1.0000	
The optimal probabilities of the prey pure strategies selection:	
P2PS_probabilities =	
0.3312 0.0204 0.1477 0.1696 0.3312	
v_opt = 0.36454	
	1
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>> [P1PS P1PS_probabilities P2PS P2PS_probabilities v_opt] = ppsr(12);	<u>^</u>
The persecutor pure strategies to be selected:	
P1PS =	
0.0600 0.0650 0.5000 0.9350 0.9400	
The optimal probabilities of the persecutor pure strategies selection:	
P1PS_probabilities =	
0.0427 0.3158 0.2829 0.3158 0.0427	
The prey pure strategies to be selected:	
P2PS =	
0 0.4000 0.6000 0.6050 1.0000	
The optimal probabilities of the prey pure strategies selection:	
P2PS_probabilities =	
0.3170 0.1855 0.0774 0.1033 0.3168	
v opt = 0.35522	
<u> </u>	-
A Start	

Figure 15. Some asymmetry of the five pure strategies of the prey by  $\alpha = 12$ 



Figure 16. By  $\alpha = 14$  the persecutor selects its five pure strategies (the zero probability must be expunged), and the prey selects its six pure strategies

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>> [P1PS P1PS	5_probabil	ities P2PS	P2PS_prob	abilities	v_opt] = ppsr(16);		
The nersed	utor nure	strategie	s to be se	lected:			
1PS =	Jacor pare	Doracegre	0 00 20 00				
0.0850	0.0900	0.5000	0.9100	0.9150			
The optima	al probabi	lities of	the persec	utor pure	strategies selecti	on:	
1PS_probabil	lities =						
0.1911	0.1653	0.2871	-0.0000	0.1653	0.1911		
The prey p	pure strat	egies to b	e selected	:			
2P5 =	0 2450	0 2500	0 6500	0 6550	1 0000		
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0.2720	0.2200	0.0079	0.0108	0.2173	0.2720		
v opt = 0.	.32075						
v_opt = 0.	.32075						
v_opt = 0. >> [P1PS P1PS	.32075 5_probabil	ities P2PS	P2PS_prob	abilities	v_opt] = ppsr(18);		
v_opt = 0.	.32075 S_probabil	ities P2PS	P2PS_prob	abilities	v_opt] = ppsr(18);		
v_opt = 0.	.32075 5_probabil	ities P2PS	P2PS_prob	abilities	v_opt] = ppsr(18);		
v_opt = 0. >> [P1PS P1PS The persec	.32075 3_probabil sutor pure	ities P2PS strategie	P2PS_prob	abilities :lected:	v_opt] = ppsr(18);		
v_opt = 0. >> [P1PS P1PS The persec P1PS =	.32075 5_probabil cutor pure	ities P2PS strategie	P2PS_prob	eabilities	v_opt] = ppsr(18);		
v_opt = 0. >> (P1PS P1PS The persec P1PS = 0.0700 The optimum	.32075 5_probabil cutor pure 0.3950	ities P2PS strategie 0.4000	P2PS_prob s to be se 0.6000	abilities lected: 0.6050	v_opt] = ppsr(18); 0.9300		
v_opt = 0. >> [P1PS P1PS The persec P1PS = 0.0700 The optime P1PS probabi	.32075 5_probabil Sutor pure 0.3950 al probabi	ities P2PS strategie 0.4000 lities of	P2PS_prob s to be se 0.6000 the persec	abilities elected: 0.6050 eutor pure	v_opt] = ppsr(18); 0.9300 strategies selecti	on:	
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v_opt = 0. >> [P1PS P1PS The persec P1PS = 0.0700 The optime P1PS_probabil 0.3224	.32075 3_probabil Sutor pure 0.3950 al probabi lities = 0.1220	ities P2P3 strategie 0.4000 lities of 0.0556	P2PS_prob s to be se 0.6000 the persec 0.0556	eabilities elected: 0.6050 sutor pure 0.1220	v_opt] = ppsr(18); 0.9300 strategies selecti 0.3224	on:	
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v_opt = 0. >> [P1PS P1PS The persec P1PS = 0.0700 The optime P1PS_probabil 0.3224 The prey p P2PS = 0	.32075 3_probabil cutor pure 0.3950 al probabi lities = 0.1220 pure strat 0.3400	ities P2PS strategie 0.4000 lities of 0.0556 egies to b 0.3450	P2PS_prob s to be se 0.6000 the persec 0.0556 e selected 0.6550	eabilities (lected: 0.6050 (sutor pure 0.1220 (). 0.6600	<pre>v_opt] = ppsr(18); 0.9300 strategies selecti 0.3224 1.0000</pre>	on:	
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v_opt = 0. >> (P1PS P1PS The persec P1PS = 0.0700 The optime P1PS_probabil 0.3224 The prey p P2PS = 0 The optime P2PS_probabil	.32075 3_probabil cutor pure 0.3950 al probabi lities = 0.1220 pure strat 0.3400 al probabi lities =	ities P2PS strategie 0.4000 lities of 0.0556 egies to b 0.3450 lities of	P2PS_prob s to be se 0.6000 the persec 0.0556 e selected 0.6550 the prey p	abilities (lected: 0.6050 utor pure 0.1220 L: 0.6600 pure strate	v_opt] = ppsr(18); 0.9300 strategies selecti 0.3224 1.0000 gies selection:		
v_opt = 0. >> [P1PS P1PS P1PS = 0.0700 The optima P1PS_probabil 0.3224 The prey p 22PS = 0 The optima 22PS_probabil 0.2671	.32075 3_probabil sutor pure 0.3950 al probabi lities = 0.1220 pure strat 0.3400 al probabi lities = 0.0949	ities P2PS strategie 0.4000 lities of 0.0556 egies to b 0.3450 lities of 0.1380	P2PS_prob s to be se 0.6000 the persec 0.0556 e selected 0.6550 the prey p 0.1380	abilities lected: 0.6050 sutor pure 0.1220 l: 0.6600 sure strate 0.0949	<pre>v_opt] = ppsr(18); 0.9300 strategies selecti 0.3224 1.0000 gies selection: 0.2671</pre>	on:	
v_opt = 0. >> [P1PS P1PS The persec 0.0700 The optime P1PS_probabil 0.3224 The prey p P2PS = 0 The optime P2PS_probabil 0.2671	.32075 3_probabil cutor pure 0.3950 al probabi lities = 0.1220 pure strat 0.3400 al probabi lities = 0.0949	ities P2PS strategie 0.4000 lities of 0.0556 egies to b 0.3450 lities of 0.1380	P2PS_prob s to be se 0.6000 the persec 0.0556 e selected 0.6550 the prey p 0.1380	eabilities lected: 0.6050 sutor pure 0.1220 l: 0.6600 pure strate 0.0949	<pre>v_opt] = ppsr(18); 0.9300 strategies selecti 0.3224 1.0000 gies selection: 0.2671</pre>	on:	
<pre>v_opt = 0. &gt;&gt; [P1PS P1PS The persec 0.0700 The optime P1PS_probabil 0.3224 The prey p P2PS = 0 The optime ?2PS_probabil 0.2671 v_opt = 0.</pre>	.32075 3_probabil cutor pure 0.3950 11 probabi 11ties = 0.1220 pure strat 0.3400 al probabi 11ties = 0.0949 .30595	ities P2PS strategie 0.4000 lities of 0.0556 egies to b 0.3450 lities of 0.1380	P2PS_prob s to be se 0.6000 the persec 0.0556 e selected 0.6550 the prey p 0.1380	eabilities elected: 0.6050 sutor pure 0.1220 l: 0.6600 sure strate 0.0949	<pre>v_opt] = ppsr(18); 0.9300 strategies selecti 0.3224 1.0000 gies selection: 0.2671</pre>	on:	

Figure 17. Pure strategies and their probabilities by  $\alpha = 16$  and  $\alpha = 18$ 



Figure 18. Symmetrization of the six pure strategies for their selection Figure 19. Appearance of the seven pure strategies for their selection by  $\alpha = 20$  by  $\alpha = 28$ 

It ought to be underlined, that the persecutor — prey systems with the optimal probability  $v_{opt} < \frac{1}{2}$  of the prey strike are out of practical interest. Then the case on the figure 12 is nearly the worst, where both the competitors should select the three pure strategies with the corresponding optimal probabilities.

The being determined within the module ppsr optimal probabilities may be easily practiced [11 - 15] by the prey or the persecutor, if it applies the earlier developed MATLAB 7.0.1 program module opr2 (figure 20, [16, 17]). It is sufficient for the prey (or the persecutor) to type in the MATLAB 7.0.1 Command Window the line with the six arguments of this module, knowing the number G of the future shots on itself (or the future shots on the target), and run this line by pressing the enter key, as it is shown on the figure 21. The module opr2 will compute the vector

$$\mathbf{V} = \begin{bmatrix} v_1 & v_2 & \dots & v_G \end{bmatrix}$$
(26)

of the persecutor payoffs  $\{v_j\}_{j=1}^G$ , where  $v_j$  is the kernel (2) value in the shot j by  $j = \overline{1, G}$ , and actually their average is converging to  $v_{opt}$  as the number of the shots is tending to infinity:

$$\lim_{j \to \infty} \frac{1}{G} \sum_{j=1}^{G} v_j = \frac{1}{G} \lim_{j \to \infty} \sum_{j=1}^{G} v_j = v_{opt}$$
(27)

Besides, in the MATLAB 7.0.1 Command Window there are returned the hints of what pure strategy to be selected, and the relative deviation

$$\delta_{v} = \frac{\frac{1}{G} \sum_{j=1}^{G} v_{j} - v_{opt}}{v_{opt}}$$
(28)

of the averaged persecutor payoff.



Figure 20. The starting part of the program module opr2 code in MATLAB 7.0.1 M-file Editor



Figure 21. The example of running the program module opr2 code with G = 8 and some kernel (2) in the matrix form

Some examples of the module opr2 application are shown below on the figures 22 - 34, and the mean probability of striking the prey dependence is plotted on the figures 35 - 36. Here it must be marked, that the program module opr2 does not require the program module ppsr running before or later, as it works fully without assistance. But instead of putting there the kernel (2) in the matrix form as the payoff matrix, there may be put just the corresponding submatrix, which lines correspond to the optimal strategy spectrum of the persecutor, and columns correspond to the optimal strategy spectrum of the prey.

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>> [Payoff_1] = opr2(P, 8, 1, 0, 0, 0)		<u> </u>
The relative deviation of the averaged payoff of the first player is 0.0072363		
Payo11_1 =       0.82796705280245       0.85440909943717       0.85271173552118       0.82796705280245       0.85440909943717       0.85271173552118         >> [Payoff_1] = opr2(P, 8, 1, 0, 0, 0)	0.85440909943717	0.85440909943717
The relative deviation of the averaged payoff of the first player is $0.011417$ Payoff_1 =		
0.85440909943717 0.82796705280245 0.85440909943717 0.85271173552118 0.85440909943717 0.85440909943717 >> [Payoff_1] = opr2(P, 8, 1, 0, 0, 0)	0.85440909943717	0.85440909943717
The relative deviation of the averaged payoff of the first player is $0.0073449$ Payoff_1 =		
0.85440909943717 0.85271173552118 0.83002990648014 0.83002990648014 0.85271173552118 0.85440909943717 >> [Payoff_1] = opr2(P, 8, 1, 0, 0, 0)	0.85271173552118	0.85271173552118
The relative deviation of the averaged payoff of the first player is $4.5992e-005$ Payoff 1 =		
0.82796705280245 0.85440909943717 0.83002990648014 0.83002990648014 0.85271173552118 0.83002990648014 >> [Payoff_1] = opr2(P, 8, 1, 0, 0, 0)	0.85271173552118	0.85271173552118
The relative deviation of the averaged payoff of the first player is -0.0030176		
Payoff_1 = 0.85440909943717 0.83002990648014 0.85271173552118 0.83002990648014 0.83002990648014 0.83002990648014	0.83002990648014	0.85271173552118
4 Start		

Figure 22. The vectors (26) by G = 8 and the same kernel (2) in the matrix form

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> [Payoff_1] = opr2(P, 10, 1, 0, 0)
The relative deviation of the averaged payoff of the first player is -0.00326 Payoff_1 = Columns 1 through 8 0.82796705280245 0.82796705280245 0.82796705280245 0.85440909943717 0.85440909943717 0.82796705280245 0.82796705280245 columns 9 through 10 0.85440909943717 0.85440909943717 >> [Payoff_1] = opr2(P, 12, 1, 0, 0, 0)
<pre>The relative deviation of the averaged payoff of the first player is -0.0078659 Payoff_1 = Columns 1 through 8 0.83002990648014 0.85271173552118 0.85271173552118 0.83002990648014 0.85271173552118 0.82796705280245 Columns 9 through 12 0.82796705280245 0.82796705280245 0.82796705280245 &gt;&gt; [Payoff_1] = opr2(P, 14, 1, 0, 0, 0)</pre>
<pre>The relative deviation of the averaged payoff of the first player is 0.0017578 Payoff_1 = Columns 1 through 8 Columns 1 through 8 0.85440909943717 0.83002990648014 0.85271173552118 0.82796705280245 0.85271173552118 0.82796705280245 0.83002990648014 Columns 9 through 14 0.85440909943717 0.85440909943717 0.85271173552118 0.82796705280245 0.82796705280245 0.85271173552118 &gt;&gt; [Payoff_1] = opr2(P, 18, 1, 0, 0, 0)</pre>
The relative deviation of the averaged payoff of the first player is 0.0035598 Payoff_1 = Columns 1 through 8 0.85440909943717 0.85440909943717 0.85271173552118 0.82796705280245 0.82796705280245 0.82796705280245 0.85440909943717
<pre>columms 9 through lb columns 9 through lb 0.85440909943717 0.85440909943717 0.83002990648014 0.83002990648014 0.83002990648014 0.85271173552118 0.8544090943717 columns 17 through l8 columns 17 through l8 &gt;&gt; [Payoff_1] = opr2(P, 20, 1, 0, 0, 0) &gt;&gt; [Payoff_1] = opr2(P, 20, 1, 0, 0, 0)</pre>
The relative deviation of the averaged payoff of the first player is 0.0013397 Payoff 1 =

Figure 23. The vectors (26) by  $G \in \{10, 12, 14, 18\}$  and the same kernel (2) in the matrix form

271173552118 271173552118	71173552118 271173552118	71173552118 271173552118 296705280245	71173552118 271173552118 796705280245 440909943717	71173552118 771173552118 796705280245 440909943717 002990648014	71173552118 771173552118 796705280245 440909943717 002990648014	71173552118 771173552118 796705280245 440909943717 002990648014 002990648014	71173552118 771173552118 796705280245 440909943717 002990648014 002990648014 002990648014	71173552118 771173552118 796705280245 140909943717 102990648014 002990648014 002990648014 002990648014 002990648014 796705280245	71173552118 771173552118 796705280245 440909943717 140909943717 002990648014 002990648014 002990648014 002990648014 002990648014 796705280245
.173552118 0.85271 .990648014 0.85271	.173552118 0.85271 .990648014 0.85271	.173552118 0.85271 :990648014 0.85271 :173552118 0.82796	.173552118 0.85271 :990648014 0.85271 :173552118 0.82796 :705280245 0.85440	.173552118 0.85271 :990648014 0.85271 :173552118 0.82796 5705280245 0.85440 :990648014 0.83002	.173552118 0.85271 :990648014 0.85271 :173552118 0.82796 5705280245 0.85440 :990648014 0.83002	.173552118 0.85271 :990648014 0.85271 :173552118 0.82796 5705280245 0.85440 :990648014 0.83002 :990648014 0.83002	.173552118 0.85271 :990648014 0.85271 ;73552118 0.82796 ;705280245 0.85440 :990648014 0.83002 :990648014 0.83002 :90943717 0.83002	.173552118 0.85271 :990648014 0.85271 :73552118 0.82796 :705280245 0.85446 :990648014 0.83002 :990648014 0.83002 :705280245 0.83002 :173552118 0.83002 :173552118 0.83002 :705280245 0.83002 :705280245 0.83002	.173552118 0.85271 :990648014 0.85271 :73552118 0.82796 ;705280245 0.85440 :990648014 0.83002 :990648014 0.83002 :705280245 0.83002 :173552118 0.83002 :705280245 0.83002 :705280245 0.83002
943717 0.8300299064	943717 0.8300299064	943717 0.8300299064 1552118 0.852711735	943717 0.8300299064 1552118 0.8527117355 1943717 0.8279670526	943717 0.8300299064 1552118 0.8527117355 943717 0.8279670528 1552118 0.8300299064	943717 0.8300299064 552118 0.8527117355 943717 0.8279670528 1552118 0.8300299064	943717 0.8300299064 552118 0.8527117355 943717 0.8279670528 552118 0.8300299064 552118 0.8300299064	943717 0.8300299064 552118 0.8527117355 1943717 0.8279670528 1552118 0.8300299064 1552118 0.8300299064 1943717 0.8544090997	943717 0.8300299064 552118 0.8527117355 943717 0.85279670528 552118 0.8300299064 552118 0.8300299064 552118 0.8544090994 552118 0.8527117355 648014 0.8527117355 552118 0.8527117355 552118 0.85279670528	943717 0.8300299064 552118 0.8527117355 943717 0.8279670528 552118 0.8300299064 552118 0.8300299064 1552118 0.85744090994 1552118 0.857717355 1648014 0.857717355 1548014 0.85779670528 1552118 0.85779670528
		118 0.852711735521	118 0.852711735521 245 0.85440909943	118 0.852711735521 245 0.854409099437 214 0.852711735521	118 0.852711735521 245 0.85479099437 314 0.852711735521	<pre>118 0.852711735521 245 0.85440909437 314 0.852711735521 314 0.852711735521 314 0.854409099437</pre>	<pre>18 0.852711735521 345 0.85440909437 314 0.852711735521 314 0.852711735521 118 0.852711735521</pre>	<pre>[18 0.852711735521 245 0.85440909437 114 0.852711735521 118 0.852711735521 118 0.852711735521 717 0.830029906480 114 0.852711735521 014 0.852711735521 014 0.852711735521</pre>	<ul> <li>118 0.852711735521</li> <li>145 0.85440909437</li> <li>114 0.852711735521</li> <li>118 0.852711735521</li> <li>118 0.852711735521</li> <li>119 0.852711735521</li> <li>114 0.852711735521</li> <li>114 0.852711735521</li> </ul>
	0041979	0041979 4 D.85271173552118	0041979 1 0.85271173552118 5 0.82796705280245	0041979 1 0.85271173552118 5 0.82796705280245 7 0.83002990648014	041979 t 0.85271173552118 5 0.82796705280245 7 0.83002990648014 2027331	041979 (0.85271173552118 (0.85271173552118 ) 0.82796705280245 0.83002990648014 00.83002990648014 0027331 0.83002990648014	041979 1041979 10.85271173552118 0.82796705280245 10.83002990648014 00.83002990648014 10.83002990648014 10.83002990648014 10.85271173552115	1041979 1041979 10.852795705280245 10.83002990648014 10.83002990648014 10.83002990648014 10.83002990648014 10.83002990648014 10.83002990648014 10.83002990648014 10.83002990648014	041979 1041979 1 0.85271173552118 0 0.82796705280245 0 0.83002990648014 0 0.83002990648014 0 0.83002990648014 0 0.83002990648014 1 0.83002990648014 1 0.83002990648014 1 0.83002990648014
	st player is 0.000	st player is 0.000 0.83002990648014	st player is 0.000 0.83002990648014 0.82796705280245	st player is 0.000 0.83002990648014 0.82796705280245 0.85440909943717	st player is 0.000 0.83002990648014 0.82796705280245 0.85440909943717 0.85440909943717	st player is 0.000 0.83002990648014 0.82796705280245 0.85440909943717 0.85440909943717 st player is -0.00	st player is 0.000 0.83002990648014 0.82796705280245 0.85440909943717 0.85440909943717 0.85440909943717 0.85271173552118	<pre>st player is 0.000 0.83002990648014 0.85796705280245 0.85440909943717 0.85440909943717 0.85440909943717 0.85271173552118 0.85271173552118 0.85271173552118 0.83002990648014</pre>	<pre>st player is 0.000 0.83002990648014 0.82796705280245 0.85440909943717 0.85440909943717 0.85440909943717 0.8541173552118 0.85271173552118 0.85271173552118 0.85271173552118 0.85271173552118</pre>
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	<pre>Dff_l] = opr2(P, 25, 1, 0, 0, 0) celative deviation of the averagec</pre>	<pre>off_l] = opr2(P, 25, 1, 0, 0, 0) relative deviation of the averaged 1 = 1 = 1 + 173552118 40009943717 0.85271173552118 ns 9 through 16</pre>	<pre>Dff_l] = opr2(P, 25, 1, 0, 0, 0) celative deviation of the averaged 1 = 1 = 40909943717 0.85271173552118 as 9 through 16 796705280245 0.82796705280245 as 17 through 24</pre>	<pre>&gt;&gt;ff_l] = opr2(P, 25, 1, 0, 0, 0) &gt;telative deviation of the averaged 1 = 1 = 1 = 0.85271173552118 40909943717 0.85271173552118 as 9 through 16 796705280245 0.82796705280245 as 17 through 24 440909943717 0.82796705280245 ar 15 211173552118 &gt;&gt;tf_l] = opr2(P, 40, 1, 0, 0, 0)</pre>	<pre>off_l] = opr2(P, 25, 1, 0, 0, 0) relative deviation of the averaged 1 = 1 = 440909943717 0.85271173552118 ns 9 through 16 796705280245 0.82796705280245 ns 17 through 24 440909943717 0.82796705280245 ns 17 through 24 440909943717 0.82796705280245 ns 17 through 24 the averaged of the averaged of the averaged the averaged of the averaged t</pre>	<pre>Dif1] = opr2(P, 25, 1, 0, 0, 0) 1 = 1 = 1 = 1 = 1 = 1 = 1 = 1 = 1 = 1 =</pre>	<pre>Dif1] = opr2(P, 25, 1, 0, 0, 0)</pre>	<pre>off_l] = opr2(P, 25, 1, 0, 0, 0) relative deviation of the averaged a 1 = 44090943717 0.85271173552118 as 9 through 16 796705280245 0.82796705280245 as 17 through 24 440909943717 0.82796705280245 as 17 through 24 400, 0, 0, 0, 0) off_l] = opr2(P, 40, 1, 0, 0, 0) aff_l] = opr2(P, 40, 1, 0, 0, 0) aff_l] = opr2(P, 40, 1, 0, 0, 0) aff_l] = opr2(P, 40, 1, 0, 0, 0) off_l] = opr2(P, 40, 1, 0, 0, 0) off_l] = opr2(P, 40, 1, 0, 0, 0) aff_l] = opr2(P, 40, 1, 0, 0, 0) aff_l] = opr2(P, 40, 1, 0, 0, 0) aff_l] = opr2(P, 50, 1, 0, 0, 0) off_l] = opr2(P, 50, 1, 0, 0, 0) off_l] = opr2(P, 50, 1, 0, 0, 0)</pre>	<pre>Difl] = opr2(P, 25, 1, 0, 0, 0)  relative deviation of the averaged 1 = 1</pre>
<pre>     I through 8     10 brough 8     10 brough 8     10 brough 16     10 brough 17     10 brough 16     10 brough 17     10 brord 17     10 brord 17     10 brord 17     10 brord 17     10</pre>	796705280245       0.82796705280245       0.82796705280245       0.82796705280245       0.82796705280245       0.85440909943717       0.85440909943717       0.85440909943717       0.85440909943717       0.85440909943717       0.85741173552118       0.83002990648014       0.830002990648014       0.830002990648014       0.830002990648014       0.830002990648014       0.830002990648014 <t< td=""><td>44090943717 0.82796705280245 0.8544090943717 0.8544090943717 0.83002990648014 0.85271173552118 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.85440909943717 0.83002990648014 0.85440909943717 0.83002990648014 0.85740909943717 0.85440909943717 0.85440909943717 0.83002990648014 0.85740909943717 0.85440909943717 0.85271173552118 0.83002990648014 0.83002990648014 0.85271173552118 0.83002990648014 0.83002990648014 0.85271173552118 0.83002990648014 0.83002990648014 0.55271173552118 0.83002990648014 0.83002990648014 0.55271173552118 0.83002990648014 0.83002990648014 0.55271173552118 0.83002990648014 0.83002990648014 0.55271173552118 0.83002990648014 0.83002990648014 0.83002990648014 0.85271173552118 0.83002990648014 0.830009993777 0.8340909993777 0.83002990648014 0.830009993777 0</td><td><pre>c=lative deviation of the averaged payoff of the first player is -0.0027331 1 = 1 = 1 = 1 = 1 = 1 = 1 = 1 = 1 = 1</pre></td><td><pre>ns 1 through 8 796705280245 0.82796705280245 0.85440909943717 0.83002990648014 0.85440909943717 0.85440909943717 0.83002990648014 ns 9 through 16 271173552118 0.83002990648014 0.83002990648014 0.85271173552118 0.85271173552118 0.83002990648014 0.83002990648014 ns 17 through 24 ns 17 through 24 796705280245 0.85440909943717 0.82796705280245 0.85440909943717 0.83002990648014 0.83002990648014 35 through 24 25 through 22 </pre></td><td>us 9 curcudu 10 271173552118 0.83002990648014 0.83002990648014 0.85271173552118 0.85271173552118 0.82796705280245 0.83002990648014 ns 17 through 24 17 55705280245 0.85440909943717 0.85440909943717 0.82796705280245 0.85440909943717 0.83002990648014 0.85271173552118 0.83002990648014</td><td>un di concessi di concessi 255502580245 di concessi di 25 di concessi d</td><td></td><td>us 35 curcueur au 002990648014  0.83002990648014  0.85271173552118  0.83002990648014  0.83002990648014  0.85271173552118  0.85440909943717  0.82796705280245 Diff_l] = opr2(P, 50, 1, 0, 0, 0)</td><td>us 33 curcular</td></t<>	44090943717 0.82796705280245 0.8544090943717 0.8544090943717 0.83002990648014 0.85271173552118 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.85440909943717 0.83002990648014 0.85440909943717 0.83002990648014 0.85740909943717 0.85440909943717 0.85440909943717 0.83002990648014 0.85740909943717 0.85440909943717 0.85271173552118 0.83002990648014 0.83002990648014 0.85271173552118 0.83002990648014 0.83002990648014 0.85271173552118 0.83002990648014 0.83002990648014 0.55271173552118 0.83002990648014 0.83002990648014 0.55271173552118 0.83002990648014 0.83002990648014 0.55271173552118 0.83002990648014 0.83002990648014 0.55271173552118 0.83002990648014 0.83002990648014 0.83002990648014 0.85271173552118 0.83002990648014 0.830009993777 0.8340909993777 0.83002990648014 0.830009993777 0	<pre>c=lative deviation of the averaged payoff of the first player is -0.0027331 1 = 1 = 1 = 1 = 1 = 1 = 1 = 1 = 1 = 1</pre>	<pre>ns 1 through 8 796705280245 0.82796705280245 0.85440909943717 0.83002990648014 0.85440909943717 0.85440909943717 0.83002990648014 ns 9 through 16 271173552118 0.83002990648014 0.83002990648014 0.85271173552118 0.85271173552118 0.83002990648014 0.83002990648014 ns 17 through 24 ns 17 through 24 796705280245 0.85440909943717 0.82796705280245 0.85440909943717 0.83002990648014 0.83002990648014 35 through 24 25 through 22 </pre>	us 9 curcudu 10 271173552118 0.83002990648014 0.83002990648014 0.85271173552118 0.85271173552118 0.82796705280245 0.83002990648014 ns 17 through 24 17 55705280245 0.85440909943717 0.85440909943717 0.82796705280245 0.85440909943717 0.83002990648014 0.85271173552118 0.83002990648014	un di concessi di concessi 255502580245 di concessi di 25 di concessi d		us 35 curcueur au 002990648014  0.83002990648014  0.85271173552118  0.83002990648014  0.83002990648014  0.85271173552118  0.85440909943717  0.82796705280245 Diff_l] = opr2(P, 50, 1, 0, 0, 0)	us 33 curcular
<pre>     through 8     through 8     through 8     through 1         0.65271173552118 0.6544000943717 0.8300290648014 0.65271173552118 0.65271173552118 0.82796705280245         0.82796705280245 0.82440909943717 0.82796705280245 0.82796705280245 0.82796705280245         0.82796705280245 0.82796705280245 0.82796705280245 0.82796705280245 0.85440909943717         0.82796705280245 0.82440909943717 0.83002990648014 0.85271173522118 0.83002990648014 0.83002990648014         0.83002990648014 0.83002990648014 0.83002990648014 0.8571173522118         0.83002990648014 0.83002990648014 0.83002990648014 0.8571173522118         0.83002990648014 0.83002990648014 0.83002990648014 0.8571173522118         0.83002990648014 0.83002990648014 0.85771173522118 0.83002990648014         0.83002990648014 0.83002990648014 0.85771173552118 0.85440909943717 0.85440909943717         0.83002990648014 0.83002990648014 0.85771173552118 0.85740909943717 0.85440909943717         0.83002990648014 0.83002990648014 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   0.83002990648014 0.83002990648014         0.830</pre>	796705200245 0.82796705280245 0.82796705280245 0.82796705280245 0.82796705280245 0.82440909943717 0.82796705280245 0.85440909943717 0.83002990648014 0.85071173552118 0.83002990648014 0.85771173552118 0.83002990648014 0.85070873717 0.85440909943717 0.85440909943717 0.83002990648014 0.8507087290648014 0.85070890648014 0.8507087290648014 0.85070890943717 0.85440909943717 0.85440909943717 0.85440909943717 0.85440909943717 0.83002990648014 0.850708290648014 0.850708290648014 0.850708290648014 0.850708290648014 0.850708290648014 0.850708909943717 0.85040909943717 0.85040909943717 0.85040909943717 0.85040909943717 0.83002990648014 0.850708173552118 0.83002990648014 0.85071173552118 0.83002990648014 0.85071173552118 0.83002990648014 0.85071173552118 0.83002990648014 0.85071173552118 0.83002990648014 0.85071173552118 0.83002990648014 0.85071173552118 0.83002990648014 0.85071173552118 0.83002990648014 0.85071173552118 0.83002990648014 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0.82796705280245 0.85440909943717 0.83002990648014 0.85440909943717 0.85440909943717 0.83002990648014 ns 9 through 16 271173552118 0.83002990648014 0.83002990648014 0.85271173552118 0.85271173552118 0.85271173552118 0.83002990648014 ns 17 through 24 796705280245 0.85440909943717 0.82796705280245 0.85440909943717 0.83002990648014 0.83002990648014 ns 25 through 32 ns 25 through 32</pre>	a surrowy to a currowy of a construction of a construction of a construction of a currowy of a construction of a constru	796705280245 0.85440909943717 0.85440909943717 0.82796705280245 0.85440909943717 0.83002990648014 0.85271173552118 0.83002990648014 a 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.83002990648014 0.85271173552118 0.83002990648014 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Figure 24. The vectors (26) by  $G \in \{20, 25, 40\}$  and the same kernel (2) in the matrix form

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<pre>&gt;&gt; [Payoff_1] = opr2(P, 50, 1, 0, 0, 0)</pre>							•
The relative deviation of the average	ed payoff of the fir	st player is 0.0027	72 69				
Payoff_1 = Columns 1 through 8							
0.83002990648014 0.85271173552118	0.82796705280245	0.85440909943717	0.85440909943717	0.82796705280245	0.85440909943717	0.85271173552118	
Columns 9 through 16 0.85440909943717 0.85440909943717	0.83002990648014	0.83002990648014	0.85271173552118	0.83002990648014	0.83002990648014	0.85271173552118	
Columns 17 through 24 0.83002990648014 0.85440909943717	0.85271173552118	0.85271173552118	0.82796705280245	0.82796705280245	0.85440909943717	0.83002990648014	
Columns 25 through 32 0.85271173552118 0.82796705280245	0.85271173552118	0.85271173552118	0.82796705280245	0.83002990648014	0.83002990648014	0.85271173552118	
Columns 33 through 40							
0.85271173552118 0.85271173552118 Columns 41 through 48	0.85271173552118	0.85271173552118	0.85271173552118	0.82796705280245	0.83002990648014	0.83002990648014	Γ
0.83002990648014 0.85440909943717	0.85271173552118	0.85271173552118	0.85271173552118	0.82796705280245	0.85271173552118	0.85271173552118	
Columns 49 through 50							
0.85271173552118 0.85271173552118 >> [Pavoff 1] = obr2(P. 100. 1. 0. 0.							
The relative deviation of the average	ed payoff of the fir	st player is 0.0015	5662				
Payoff_1 =							
Columns 1 through 8 0.83002990648014 0.83002990648014	0.85271173552118	0.85271173552118	0.85271173552118	0.85271173552118	0.85271173552118	0.85271173552118	
Columns 9 through 16							
0.82796705280245 0.85440909943717	0.85271173552118	0.85271173552118	0.82796705280245	0.83002990648014	0.83002990648014	0.85440909943717	
Columns 17 through 24 n eannachteanta n eannachteanta	0 0001173663110	0 0200000640044	0 06071173660110	0 02006706200346	0 05071170550110	0 0200000640014	
Columns 25 through 32	OTT 2000) TT, 200 O	LT00L006670000.0	OTT 2000) TT / 200 O	CE 2002CO/06/20.0		£700£00 <i>663</i> 00000.0	
0.85271173552118 0.82796705280245	0.83002990648014	0.85271173552118	0.83002990648014	0.85271173552118	0.82796705280245	0.85440909943717	
Columns 33 through 40 0.85440909943717 0.82796705280245	0.83002990648014	0.85440909943717	0.85271173552118	0.85271173552118	0.82796705280245	0.85271173552118	
Columns 41 through 48							
0.85271173552118 0.82796705280245 Columns 49 through 56	0.85440909943717	0.85271173552118	0.85271173552118	0.82796705280245	0.85440909943717	0.85271173552118	
0.85271173552118 0.85271173552118	0.85271173552118	0.82796705280245	0.82796705280245	0.82796705280245	0.83002990648014	0.85440909943717	
Columns 57 through 64							
U.034/11/3334110 U.04/90/U3400443 Columns 65 through 72	£TN0£906670000.0	£TN0£0N667NNc0.N	/ T/ CTAANANTTOO'N	/ T/ CIARDEDIII	#Tno#0n667nnco.n	#Tno#on667nnco.n	
0.85271173552118 0.85440909943717 Columns 73 through 80	0.82796705280245	0.83002990648014	0.83002990648014	0.85440909943717	0.85440909943717	0.85440909943717	Þ
▲ Starr							1 `
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Figure 25. The vector (26) by G = 50 and the same kernel (2) in the matrix form, and the first portion of the vector (26) by G = 100

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□         isotropy         i	Directory: E:WATLAB7p0p1W 0.82796705280245	vork 🚽 .	ے <b>اگ</b> 0.83002990648014	0.85440909943717	0.85440909943717	0.85440909943717
Columns 73 through 80 0.85271173552118 0.82796705280245	0.83002990648014	0.85271173552118	0.83002990648014	0.83002990648014	0.85440909943717	0.82796705280245
Columns of through op 0.85440909943717 0.82796705280245 Columns of +brough of	0.85271173552118	0.85440909943717	0.82796705280245	0.82796705280245	0.85271173552118	0.83002990648014
Columns of through 50 0.8544090943717 0.85271173552118 Columns 97 through 100	0.85271173552118	0.85271173552118	0.85271173552118	0.85271173552118	0.85271173552118	0.85271173552118
0.82796705280245 0.83002990648014 >> [Payoff_1] = opr2(P, 150, 1, 0, 0, 0)	0.85271173552118	0.82796705280245				
The relative deviation of the average	d payoff of the fir	st player is -0.000	125192			
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0.8544090943717 0.8544090943717	0.83002990648014	0.85440909943717	0.83002990648014	0.83002990648014	0.83002990648014	0.85271173552118
Columns 1/ through 24 0.85271173552118 0.83002990648014	0.85440909943717	0.82796705280245	0.85440909943717	0.85440909943717	0.83002990648014	0.85440909943717
Columns 25 through 32 0.83002990648014 0.83002990648014	0.85440909943717	0.85440909943717	0.83002990648014	0.82796705280245	0.83002990648014	0.83002990648014
Columns 33 through 40						
Columns 41 through 48	0TT7CC2/TT/7C2.0	) T) 246606044020	C#2007C0/96/70.0	C#2002C0/96/20.0	) T) 246606044020	0TT7CC2/TT/7CQ.0
0.85440909943717 0.83002990648014	0.85440909943717	0.83002990648014	0.83002990648014	0.85440909943717	0.82796705280245	0.85271173552118
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Columns 57 through 64		LIDOLOD <i>667</i> 0000.0				
0.83002990648014 0.85440909943717	0.85440909943717	0.85440909943717	0.83002990648014	0.83002990648014	0.85440909943717	0.83002990648014
Columns 65 through 72 0.85271173552118 0.85440909943717	0.85440909943717	0.85440909943717	0.83002990648014	0.82796705280245	0.83002990648014	0.83002990648014
Columns 73 through 80						
0.85271173552118 0.82796705280245 Columna 81 through 88	0.85271173552118	0.82796705280245	0.83002990648014	0.85271173552118	0.85440909943717	0.83002990648014
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Columns 89 through 96						
0.85271173552118 0.82796705280245	0.83002990648014	0.85440909943717	0.82796705280245	0.83002990648014	0.83002990648014	0.83002990648014
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Columns 105 through 112						
0.85271173552118 0.82796705280245 Columns 113 through 120	0.85271173552118	0.82796705280245	0.85271173552118	0.83002990648014	0.85271173552118	0.82796705280245
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Figure 26. The second portion of the vector (26) by G = 100 and the same kernel (2) in the matrix form, and the first portion of the vector (26) by G = 150

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|                                  |                                     | 0.82796705280245   | 0.85271173552118   | 0.83002990648014   | 0.83002990648014   |  |  |   
  | 0.85271173552118   | 0 054400000457  |   | 0.85271173552118   | 0.82796705280245  |  
   
  | 0.85271173552118                                  | 0.85271173552118  |  | 0.82796705280245  | 0.85271173552118  
   
  | 0 060011170660110  |  | 0.85271173552118  | 0.82796705280245  |  
   | 0.85271173552118   | 0.82796705280245   
                                   | 0 2200200642014  | 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | | | | | | |
|                                  |                                     | 0.85271173552118   | 0.83002990648014   | 0.82796705280245   | 0.85271173552118   |  |  |   
  | 0.82796705280245   | 71264000000000000000000000000000000000000   |   | 0.85440909943717   | 0.8544090943717   |  
   
  | 0.85440909943717                                  | 0.85271173552118  |  | 0.85271173552118  | 0.85271173552118  
   
  | 0 06371173663110   |  | 0.85271173552118  | 0.85271173552118  |  
   | 0.83002990648014   | 0.85271173552118   
                                   | 0 8300200648014  |   | | | | | | |
|                                  |                                     | 0.85271173552118   | 0.83002990648014   | 0.85440909943717   | 0.85271173552118   | 0.82796705280245   |  |   
  | 0.83002990648014   | 0 00706700746   |   | 0.85271173552118   | 0.83002990648014  |  
   
  | 0.85271173552118                                  | 0.85271173552118  |  | 0.85271173552118  | 0.85271173552118  
   
  | 0 06071173660110   |  | 0.85440909943717  | 0.82796705280245  |  
   | 0.85271173552118   | 0.85271173552118   
                                   | 0 877067705780745  |   | | | | | | |
|                                  | -<br>-                              | 0.82796705280245   | 0.83002990648014   | 0.82796705280245   | 0.85271173552118   | 0.82796705280245   | t1e-005  |   
  | 0.83002990648014   | 0 0000000000000000000000000000000000000   | 110000000000000000000000000000000000000   | 0.83002990648014   | 0.82796705280245  |  
   
  | 0.83002990648014                                  | 0.85440909943717  |  | 0.85271173552118  | 0.85440909943717  
   
  | 0 054400000045717  |  | 0.85440909943717  | 0.85440909943717  |  
   | 0.85271173552118   | 0.82796705280245   
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|                                  | vork 🚽                              | 0.85440909943717   | 0.83002990648014   | 0.85271173552118   | 0.85271173552118   | 0.82796705280245   | st player is -1.334  |   
  | 0.83002990648014   | 7176900000000000000000000000000000000000  |   | 0.83002990648014   | 0.85271173552118  |  
   
  | 0.85271173552118                                  | 0.83002990648014  |  | 0.85271173552118  | 0.83002990648014  
   
  | 0 03706706300345   |  | 0.82796705280245  | 0.85440909943717  |  
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                                   | 0 85071170550118   |   | | | | | | |
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  | 0.85440909943717                                  | 0.85440909943717  |  | 0.82796705280245  | 0.82796705280245  
   
  | 0 06071173660110   |  | 0.82796705280245  | 0.83002990648014  |  
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| ndow <u>H</u> elp                | -   🎁 💅   😵   Current               | 120<br>0.82796705280245  | 0.83002990648014   | , 136<br>0.85271173552118  | 144<br>0.85271173552118  | . 150<br>0.85271173552118<br>P, 200, 1, 0, 0, 0)   | tion of the average  |   
  | :<br>0.85271173552118  | .6<br>0 02706706200246  | 24  | 0.85440909943717   | 32<br>0.85271173552118  | 40   
   
  | 0.82796705280245<br>48                            | 0.82796705280245  | 56   | 0.85271173552118<br>64  | 0.82796705280245  
   
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  | Columns 65 through   | Columns 73 through   | 0.82796705280245  | Columns 81 through<br>0.85271173552118  | Columns 89 through   
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Figure 27. The second portion of the vector (26) by G = 150 and the same kernel (2) in the matrix form, and the first portion of the vector (26) by G = 200

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Columns 41 through 48 0.85271173552118 0.82796705280245	0.85440909943717	0.83002990648014	0.85440909943717	0.85271173552118	0.85271173552118	0.85271173552118
Columns 49 through 56						
U.852/11/3552118 U.852/11/3552118 Columns 57 through 64	0.827957U525U745728.U	BI12666/11/268.U	8112666/11/268.0	0112666/11/268.0	0117995/11/799.O	C#2082C0/96/28.0
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Columns 65 through 72						
U.852/11/3552118 U.852/11/3552118 Columns 73 through 80	0.852/11/358.0	C#208250/96/28.0	U.8544U9U9443717	0.852/11/3552/11	0.852/11/358.U	0.852/11/3552118
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Columns 81 through 88						
0.85271173552118 0.82796705280245	0.83002990648014	0.85440909943717	0.85440909943717	0.82796705280245	0.85271173552118	0.82796705280245
Columns 89 through 96 0.85771173552118 0.82796705280245	0.82796705280245	0.85271173552118	0.85271173552118	0.85271173552118	0.83002990648014	0.85271173552118
Columns 97 through 104						
0.82796705280245 0.83002990648014	0.85271173552118	0.85271173552118	0.82796705280245	0.85271173552118	0.85271173552118	0.82796705280245
Columns 105 through 112						
0.85440909943717 0.82796705280245	0.83002990648014	0.85271173552118	0.83002990648014	0.82796705280245	0.83002990648014	0.83002990648014
Columns 113 through 120						
0.82796705280245 0.82796705280245	0.85440909943717	0.85271173552118	0.83002990648014	0.83002990648014	0.83002990648014	0.85440909943717
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Columns 129 through 136						
0.85440909943717 0.82796705280245	0.82796705280245	0.83002990648014	0.83002990648014	0.83002990648014	0.83002990648014	0.83002990648014
Columns 137 through 144						
0.82796705280245 0.82796705280245	0.83002990648014	0.85440909943717	0.85440909943717	0.85271173552118	0.83002990648014	0.85271173552118
Columns 145 through 152						
0.83002990648014 0.83002990648014 Colimne 153 through 160	0.85271173552118	0.83002990648014	0.85271173552118	0.85440909943717	0.85440909943717	0.85271173552118
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Columns 161 through 168						
0.83002990648014 0.85271173552118	0.82796705280245	0.85440909943717	0.83002990648014	0.82796705280245	0.83002990648014	0.83002990648014
Columns 169 through 176						
0.82796705280245 0.83002990648014	0.85271173552118	0.82796705280245	0.85440909943717	0.82796705280245	0.82796705280245	0.85440909943717
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Columns 193 through 200						
0.85271173552118 0.85440909943717	0.85440909943717	0.85440909943717	0.85440909943717	0.82796705280245	0.83002990648014	0.85271173552118
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Figure 28. The second portion of the vector (26) by G = 200 and the same kernel (2) in the matrix form

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<pre>&gt;&gt; [Payoff_1] = opr2(P, 11, 1, 0, 0, 1)</pre>	•
Now the first player has selected the pure strategy x3	
Now the second player has selected the pure strategy y1	
Now the first player real payoff is 0.99639	
Now the first player has selected the pure strategy x3	
NUW TO ESCOND PIAYER TAS SElected the Strategy YI	
NOW THE LIGU PLAYER FEAL PAYOLE IS U.99009 Now the first blayer has selected the bure stratedy x46	
Now the second player has selected the pure strategy V1	
Now the first player real payoff is 0.058463	
Now the first player has selected the pure strategy x3	
Now the second player has selected the pure strategy y1	
Now the first player real payoff is 0.99639	
Now the first player has selected the pure strategy x3	
Now the second player has selected the pure strategy y1	
Now the first player real payoff is 0.99639	
Now the first player has selected the pure strategy x46	
Now the second player has selected the pure strategy yl	
Now the first player real payoff is 0.058463	
Now the first player has selected the pure strategy x3	
Now the second player has selected the pure strategy y1	
Now the first player real payoff is 0.99639	
Now the first player has selected the pure strategy x3	
Now the second player has selected the pure strategy y1	
Now the first player real payoff is 0.99639	
Now the first player has selected the pure strategy x47	
Now the second player has selected the pure strategy yl	
Now the first player real payoff is 0.052472	
Now the first player has selected the pure strategy x3	
Now the second player has selected the strategy yl	
Now the first player real payoff is U.99639	
Now the first player has selected the pure strategy x3	
Now the second player has selected the pure strategy y1	
At last, the first player real payoff is 0.99639	
The relative deviation of the averaged payoff of the first player is 0.4329	
Payoff_1 =	
Columns 1 through 8 0.99638832555360 0.99638832555360 0.05846265388925 0.99638832555360 0.99638832555360 0.05846265388925 0.99638832555360	.99638832555360
Columns 9 through 11	
0.05247227457122 $0.99638832555360$ $0.99638832555360$	
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Figure 29. The hints of what pure strategy to be selected and the vector (26) by G = 11 and another kernel (2) in the matrix form

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0.05247227457122 0.99638832555360 0.99638832555360	
<pre>&gt;&gt; [Payoff_1] = opr2(P, 23, 1, 0, 0, 1)</pre>	
Now the first player has selected the pure strategy x3	
Now the second player has selected the pure strategy yl	
Now the first player real payoff is 0.99639	
Now the first player has selected the pure strategy x46	
Now the second player has selected the pure strategy y1	
Now the first player real payori is u.usd463 Now the first player has selected the pure strategy x47	
Now the second player has selected the pure strategy y51	
Now the first player real payoff is 0.97782	
Now the first player has selected the pure strategy x3	
Now the second player has selected the pure strategy y51	
Now the first player real payoff is 0.040703	
Now the first player has selected the pure strategy x3	
Now the second player has selected the pure strategy y51 Now the first vlamer real veroff is 0 040703	
Now CHE IIISU PIAYEL FEAL PAYOLI IS ULOIO Now the first blamer has selected the nure strateon v2	
Now the fitst player has selected the pure strategy x3 Now the second mlayer has selected the pure strategy v1	
Now the first player real payoff is 0.99639	
Now the first player has selected the pure strategy x46	
Now the second player has selected the pure strategy yl	
Now the first player real payoff is 0.058463	
Now the first player has selected the pure strategy x47	
Now the second player has selected the pure strategy y51	
Now the first player real payoff is 0.97782	
Now the first player has selected the pure strategy x46	
Now the second prayer has selected the strategy yi Now the first player real payoff is 0.058463	
Now the first player has selected the pure strategy x47	
Now the second player has selected the pure strategy 722	
Now the first player real payoff is 0.41319	
Now the first player has selected the pure strategy x46	
Now the second player has selected the pure strategy yl	
Now the first player real payoff is 0.058463	
Now the first player has selected the pure strategy x47	
Now the second player has selected the pute surately yi Now the first vlaner real veroff is 0 053473	
Now the first player heat payort is could't Now the first player has selected the pure strategy X46	
Now the second player has selected the pure strategy 751	
Now the first player real payoff is 0.9698	
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Figure 30. The first portion of the hints of what pure strategy to be selected and the vector (26) by G = 23 and another kernel (2) in the matrix form

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Figure 31. The second portion of the hints of what pure strategy to be selected and the vector (26) by G = 23 and another kernel (2) in the matrix form

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>> [Payoff_1] = opr2(P, 33, 1, 0, 0, 1)	•
Now the first player has selected the pure strategy x46	
Now the second player has selected the pure strategy y1	
Now the first player real payoff is 0.058463	
Now the first player has selected the pure strategy X46	
Nuw the first blayer has selected the pute strategy you Now the first blayer real baroff is 0.9698	
Now the first player has selected the pure strategy X3	
Now the second player has selected the pure strategy y1	
Now the first player real payoff is 0.99639	
Now the first player has selected the pure strategy x46	
Now the second player has selected the pure strategy y51	
Now the first player real payoff is 0.9698	
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Now the second blayer has selected the pure strategy vis	
Now the first player real payoff is 0.9698	
Now the first player has selected the pure strategy x46	
Now the second player has selected the pure strategy y51	
Now the first player real payoff is 0.9698	
Now the first player has selected the pure strategy x3	
Now the second player has selected the pure strategy y51	
NUM THE LIEST PLAYER FEAL PAYOLL IS ULTOTOD	
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Now the first player real payoff is 0.97782	
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Now the second player has selected the pure strategy y51	
Now the first player real payoff is 0.9698	
Now the first player has selected the pure strategy x3	
Now the second player has selected the pure strategy y51	
Now the first player real payoff is 0.040703	
Now the first player has selected the strategy X+0	
Now the second player has selected the pure strategy yol Now the first blayer real baroff is 0.9608	
num the first blager has selected the nume stratedy x47	
Now the second player has selected the pure strategy 751	
Now the first player real payoff is 0.97782	
Now the first player has selected the pure strategy x3	F
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Figure 32. The first portion of the hints of what pure strategy to be selected and the vector (26) by G = 33 and another kernel (2) in the matrix form

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Now the first player has selected the pure strategy x3	
Now the second player has selected the pure strategy y51	
Now the first player real payoff is 0.040703	
Now the first player has selected the pure strategy x3	
Now the second player has selected the pure strategy y51	
Now the first player real payoff is 0.040703	
Now the first player has selected the pure strategy x3	
Now the second player has selected the pure strategy 751	
Now the first player real payoff is 0.040703	
Now the first player has selected the pure strategy x46	
Now the second player has selected the pure strategy 751	
Now the first player real payoff is 0.9698	
Now the first player has selected the pure strategy x46	
Now the second player has selected the pure strategy 751	
Now the first player real payoff is 0.9698	
Now the first player has selected the pure strategy x3	
Now the second player has selected the pure strategy 751	
Now the first player real payoff is 0.040703	
Now the first player has selected the pure strategy x46	
Now the second player has selected the pure strategy $\gamma 1$	
Now the first player real payoff is 0.058463	
Now the first player has selected the pure strategy x47	
Now the second player has selected the pure strategy 751	
Now the first player real payoff is 0.97782	
Now the first player has selected the pure strategy x46	
Now the second player has selected the pure strategy $\gamma 1$	
Now the first player real payoff is 0.058463	
Now the first player has selected the pure strategy x47	
Now the second player has selected the pure strategy 751	
Now the first player real payoff is 0.97782	
Now the first player has selected the pure strategy x46	
Now the second player has selected the pure strategy y1	
Now the first player real payoff is 0.058463	
Now the first player has selected the pure strategy x47	
Now the second player has selected the pure strategy $y_1$	
Now the first player real payoff is 0.052472	
Now the first player has selected the pure strategy x3	
Now the second player has selected the pure strategy 751	
Now the first player real payoff is 0.040703	
Now the first player has selected the pure strategy x3	
Now the second player has selected the pure strategy y51	
Now the first player real payoff is 0.040703	
Now the first player has selected the pure strategy x3	
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Figure 33. The second portion of the hints of what pure strategy to be selected and the vector (26) by G = 33 and another kernel (2) in the matrix form

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ted the pure strategy x46					
sted the pure strategy y1 ff is 0 058463					
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ET 13 0.052472					
ted the pure strategy x3 ted the pure strategy y51					
f is 0.040703					
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49932 0.9963883255360	0.96979810649932	0.04070252637372	0.96979810649932	0.96979810649932	0.04070252637372
49932 0.04070252637372	0.96979810649932	0.97781961741164	0.04070252637372	0.04070252637372	0.04070252637372
49932 0.04070252637372	0.05846265388925	0.97781961741164	0.05846265388925	0.97781961741164	0.05846265388925
637372 0.04070252637372	0.99638832555360	0.04070252637372	0.04070252637372	0.9963883255360	0.99638832555360

Figure 34. The third portion of the hints of what pure strategy to be selected and the vector (26) by G = 33 and another kernel (2) in the matrix form



Figure 35. The averaged probability of striking the prey dependence from the number of shots  $G \in \{\overline{10, 100}\}$ 

 $\mathfrak{G}$ 



Figure 36. The averaged probability of striking the prey dependence from the number of shots  $G \in \{\overline{100, 200}\}$ 

The rugged exponential probability kernel

Again check the real kernel (2) for the game convexity and concavity. The first derivative of the function (2) by the variable x is

$$\frac{\partial P(x, y)}{\partial x} = \frac{\partial}{\partial x} \left( \exp\left[ -\alpha \left( x - y \right)^2 \right] + n(x, y) \right) = \frac{\partial}{\partial x} n(x, y) - 2\alpha \left( x - y \right) \exp\left[ -\alpha \left( x - y \right)^2 \right]$$
(29)

and its second derivative is

$$\frac{\partial^2 P(x, y)}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial}{\partial x} n(x, y) - 2\alpha (x - y) \exp\left[ -\alpha (x - y)^2 \right] \right) =$$

$$= \frac{\partial^2 n(x, y)}{\partial x^2} - 2\alpha \exp\left[ -\alpha (x - y)^2 \right] + 4\alpha^2 (x - y)^2 \exp\left[ -\alpha (x - y)^2 \right] =$$

$$= 2\alpha \left( 2\alpha (x - y)^2 - 1 \right) \exp\left[ -\alpha (x - y)^2 \right] + \frac{\partial^2 n(x, y)}{\partial x^2}.$$
(30)

The first derivative of the function (2) by the variable y is

$$\frac{\partial P(x, y)}{\partial y} = \frac{\partial}{\partial y} \left( \exp\left[ -\alpha \left( x - y \right)^2 \right] + n(x, y) \right) = 2\alpha \left( x - y \right) \exp\left[ -\alpha \left( x - y \right)^2 \right] + \frac{\partial}{\partial y} n(x, y)$$
(31)

and its second derivative is

$$\frac{\partial^2 P(x, y)}{\partial y^2} = \frac{\partial}{\partial y} \left( 2\alpha (x - y) \exp\left[-\alpha (x - y)^2\right] + \frac{\partial}{\partial y} n(x, y) \right) =$$

$$= \frac{\partial^2 n(x, y)}{\partial y^2} - 2\alpha \exp\left[-\alpha (x - y)^2\right] + 4\alpha^2 (x - y)^2 \exp\left[-\alpha (x - y)^2\right] =$$

$$= 2\alpha \left(2\alpha (x - y)^2 - 1\right) \exp\left[-\alpha (x - y)^2\right] + \frac{\partial^2 n(x, y)}{\partial y^2}.$$
(32)

Again, the concavity condition  $\frac{\partial^2 P(x, y)}{\partial x^2} \leq 0$  must be true  $\forall x \in [0; 1]$  and  $\forall y \in [0; 1]$ , and, reversely, the

convexity condition  $\frac{\partial^2 P(x, y)}{\partial y^2} \ge 0$  must be true  $\forall x \in [0, 1]$  and  $\forall y \in [0, 1]$ . The concavity condition

$$2\alpha \Big(2\alpha \big(x-y\big)^2 - 1\Big) \exp\left[-\alpha \big(x-y\big)^2\right] + \frac{\partial^2 n(x,y)}{\partial x^2} \leqslant 0$$
(33)

and the convexity condition

$$2\alpha \Big(2\alpha \big(x-y\big)^2 - 1\Big) \exp\left[-\alpha \big(x-y\big)^2\right] + \frac{\partial^2 n(x,y)}{\partial y^2} \ge 0$$
(34)

may be verified numerically with as high precision as needed. The concavity of the being explored game will point at that the persecutor has the pure strategy, and the convexity will point at the prey having the pure strategy. Further, within here, it will be programmed within the updated MATLAB 7.0.1 program module ppsr. This updated module should accept minimally the two inputs, first of which is the parameter  $\alpha$ . The second input is the roughness surface n(x, y), being added to the exponential probability (1). Naturally, that by n(x, y) = 0 the updated program module ppsr2, must function as the prime original module ppsr. The screenshot of the module ppsr2 code is on the figure 37. Clearly, the precision of the convexity and concavity verification depends on the number of the input points of the roughness surface n(x, y).

The examples of the program module ppsr2 application require to have the surface n(x, y). To produce this surface artificially there has been coded a script line (figure 38), which returning had been used by plotting the figures 1 — 4. With the artificial roughness surface n(x, y) there are the 13 live results (figures 39 — 51) of having applied the

program module ppsr2 by different  $\alpha$  and amplitude of the surface n(x, y) roughness.



Figure 37. The screenshot of the module ppsr2 full code



Figure 38. The fragment of the script code, which returns the sampled surface n(x, y), what had been used by plotting the figures 1-4

With the number of the future shots G, for practicing the been determined optimal probabilities, the prey or the persecutor should apply the earlier mentioned MATLAB 7.0.1 program module opr2 by typing in the MATLAB 7.0.1 Command Window the line with the number G and running this line by the pressed enter key. The module opr2 will compute the vector (26) and the value (28), where actually (27) stays. All the hints, obviously, appear also, if they are checked with the corresponding option.

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<pre>&gt;&gt; [PIPS PIPS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(0.2, 0.001*randn(201));</pre>	
The persecutor pure strategies to be selected:	
<pre>FIPS = 0.4550 0.5000 0.5100 0.5200 The optimal probabilities of the persecutor pure strategies selection: FIPS_probabilities = 0.3209 0.0583 0.1107 0.5101</pre>	
The prey pure strategies to be selected: P2PS = 0 0.9900 0.9950 1.0000 The optimal probabilities of the prey pure strategies selection: P2PS_probabilities = 0.2492 0.2431	
$v_{-}$ opt = 0.95221	
<pre>&gt;&gt; [PIPS PIPS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(0.2, 0.002*randn(201));</pre>	
The persecutor pure strategies to be selected: PIPS =	
0.4300 0.4700 0.4850 0.4900 0.4950 0.5500 0.6050 The optimal probabilities of the persecutor pure strategies selection: PIPS_probabilities = 0.0046 0.0191 0.2126 0.3008 0.1055	
The prey pure strategies to be selected:	
<pre>F2F5 = 0 0.0050 0.0100 0.0150 0.9900 0.9950 1.0000 The optimal probabilities of the prey pure strategies selection:</pre>	
P2PS_probabilities = 0.2172 0.0659 0.1808 0.0476 0.1246 0.2256 0.1383	
$v_{opt} = 0.95292$	
Å <u>Start</u>	

Figure 39. Program module <code>ppsr2</code> application with  $\,\alpha=0.2\,$  and some low noise amplitudes

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<pre>&gt;&gt; [PIPS PIPS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(0.3, 0.002*randn(201));</pre>	1
The persecutor pure strategies to be selected:	
P1PS = 0.4600 0.5100 0.5600 0.5650	
The optimal probabilities of the persecutor pure strategies selection: P1PS_probabilities = 0.0000000000000000000000000000000000	
The prey pure strategies to be selected:	
P2PS =	
0 0.0050 0.0100 0.9950 1.0000 The optimal probabilities of the prey pure strategies selection:	
r2r5_probabilities = 0.2334 0.1883 0.3104 0.2477 0.0202 0.2334 0.1883 0.3104	
v_opt = 0.92879	
<pre>&gt;&gt; [P1PS P1PS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(0.3, 0.002*randn(201));</pre>	
The persecutor pure strategies to be selected:	
P1PS = 0.4250 0.4650 0.4750 0.5050 0.5450	
The optimal probabilities of the persecutor pure strategies selection:	
P1PS_probabilities = 0.1063 0.0242 0.1759 0.2264 0.0027 0.4644	
The prey pure strategies to be selected:	
7272 0 0.0050 0.0100 0.0250 0.9950 1.0000 The optimal probabilities of the prey pure strategies selection:	
P2PS_probabilities = 0.1636 0.2161 0.0964 0.0349 0.2457 0.2433	
$v_{obt} = 0.9301$	
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Figure 40. Program module <code>ppsr2</code> application with  $\alpha=0.3$  and the twice greater noise amplitude

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> [PIPS PIPS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(0.3, 0.003*randn(201));
The persecutor pure strategies to be selected: D.D. =
The optimal probabilities of the persecutor pure strategies selection:
PIPS_probabilities = 0.0388 0.4651 0.1679 0.1685 0.1085 0.0512 0.0388 0.4651 0.1685
The prey pure strategies to be selected:
The optimal probabilities of the prey pure strategies selection:
$r_{cr_{2}}$
$v_{o}pt = 0.93056$
<pre>&gt;&gt; [P1PS P1PS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(0.3, 0.004*randn(201));</pre>
The persecutor pure strategies to be selected: D1D4 =
PIPS_probabilities = 0.3302 0.1036 0.0127
The prey pure strategies to be selected:
rero = 0.0050 0.0100 0.9900 0.9950 1.0000 The ontiwal wrobabilities of the wee wure strateries selection:
P2PS_probabilities = 0.1853 0.0654 0.3523 0.1107
v_opt = 0.93092
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Figure 41. Program module <code>ppsr2</code> application with  $\alpha = 0.3$  and more great noise amplitudes

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>> [P1PS P1PS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(0.3, 0.	0.007*randn	:(201));		
The persecutor pure strategies to be selected: P1PS =				
0.3400 0.4100 0.4300 0.4700 0.4750 0.4950 0.5050 The optimal probabilities of the persecutor pure strategies selection:	0.5200	0.5450	0.5700	
rira_proxemilities = 0.0197 0.0039 0.0150 0.1742 0.0715 0.0051 0.2532 0.0197 0.0039 0.0150	0.0553	0.2028	0.1993	
The prey pure strategies to be selected:				
Term 0 0.0050 0.0100 0.0150 0.0250 0.9750 0.9850 The optimal probabilities of the prey pure strategies selection:	0066.0	0.9950	1.0000	
rura_proxemilities - 0.1463 0.1048 0.0501 0.1334 0.0936 0.0584 0.0490	0.0386	0.1314	0.1944	
v_opt = 0.93173				
>> [PIPS P1PS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(0.3, 0.	0.010*randn	;(201));		
The persecutor pure strategies to be selected: D1ps =				
The optimal probabilities of the persecutor pure strategies selection:	0.5850	0.6050	0.7050	
rira_proxemiiites - 0.0627 0.0972 0.4322 0.1584 0.1105 0.0578 0.0043	0.0102	0.0328	0.0338	
The prey pure strategies to be selected: P2PS =				
0 0.0050 0.0100 0.0150 0.0250 0.0300 0.9850 The optimal probabilities of the prey pure strategies selection:	0066.0	0.9950	1.0000	
rera_proxemilities - 0.3329 0.0159 0.0042 0.0007 0.0917 0.0333 0.0474	0.0749	0.3180	0.0810	
v_opt = 0.93376				
*				
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Figure 42. Program module <code>ppsr2</code> application with  $\alpha=0.3$  and some high noise amplitude

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<pre>&gt;&gt; [PIPS PIPS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(0.3, 0.010*randn(201));</pre>	
The persecutor pure strategies to be selected:	
P1PS = 0.4400 0.4550 0.4850 0.5100 0.5250 0.5600 0.5650 0.5750 The optimal probabilities of the persecutor pure strategies selection:	
P1PS_probabilities = 0.1787 0.1033 0.2644 0.1562 0.0946 0.0480 0.1000 0.0548	
The prey pure strategies to be selected:	
<pre>P2P5 = 0 0.0050 0.0100 0.0250 0.9750 0.9850 0.9900 1.0000 The optimal probabilities of the prey pure strategies selection:</pre>	
F2F5_probabilities = 0.0977 0.0825 0.1363 0.0252 0.0613 0.2724	
v_opt = 0.93386	
<pre>&gt;&gt; [PIPS PIPS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(0.3, 0.010*randn(201));</pre>	
The persecutor pure strategies to be selected:	
0.3900 0.4300 0.4350 0.4450 0.5200 0.5250 0.5300 0.5550 0.5700 The optimal probabilities of the persecutor pure strategies selection:	0.6300
PIPS_probabilities = 0.0222 0.0013 0.0446 0.1322 0.2796 0.0371 0.2234 0.1307 0.1129 0.0222 0.0013 0.0446 0.1322 0.2796 0.0371 0.2234	0.0160
The prey pure strategies to be selected: Dyps =	
0         0.0050         0.0100         0.0150         0.0300         0.9750         0.9850           The optimal probabilities of the prey pure strategies selection:         0.9750         0.9850         0.9950	1.0000
۲٬۲۰- probabilities = 0.2357 0.0331 0.0906 0.0350 0.0383 0.0437 0.0397 0.2405 0.1915	0.0519
v_opt = 0.93319	
🔦 <u>Start</u>	

Figure 43. The two repeats of the previous runs

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<pre>&gt;&gt; [P1PS P1PS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(0.5, 0.000010*randn(201));</pre>
The persecutor pure strategies to be selected:
P1P3 = 0.4950 0.5050
The optimal probabilities of the persecutor pure strategies selection: PIPS_probabilities =
The prey pure strategies to be selected: P2PS =
0 1 The optimal probabilities of the prey pure strategies selection:
P2PS_probabilities = 0.5001 0.4999
v_opt = 0.88249
<pre>&gt;&gt; [PlPS plPS_probabilities P2PS_probabilities v_opt] = ppsr2(0.5, 0.00010*randn(201));</pre>
The persecutor pure strategies to be selected:
0.4850 0.5050
The optimal probabilities of the persecutor pure strategies selection: PIPS_probabilities = 0.2548 0.7452
The prey pure strategies to be selected:
P2PS = 0 1
The optimal probabilities of the prey pure strategies selection: P2PS_probabilities = 0 5000 0 4004
v_opt = 0.88256
*
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Figure 44. Program module <code>ppsr2</code> application with  $\alpha = 0.5$  by very low noise amplitude and ten times greater one

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<pre>&gt;&gt; [PIPS PIPS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(0.5, 0.0010*randn(201));</pre>	1
The persecutor pure strategies to be selected: P1PS = 0.4950 0.5450	
The optimal probabilities of the persecutor pure strategies selection: P1FS_probabilities = 0.9249 0.0751	
The prey pure strategies to be selected: P2PS =	
0 1 The optimal probabilities of the prey pure strategies selection: P2PS_probabilities = 0.4903 0.5097	
$v_{obt} = 0.88306$	
<pre>&gt;&gt; [PIPS PIPS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(0.5, 0.0020*randn(201));</pre>	
The persecutor pure strategies to be selected: PIPS =	
0.4700 0.5200 The optimal probabilities of the persecutor pure stratedies selection:	
P1FS_probabilities =	
The prey pure strategies to be selected: P2PS =	
0 1 The optimal probabilities of the prey pure strategies selection: P2FS_probabilities =	
0.4842 0.5158 V opt = 0.88371	
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>> [P1PS P1PS_probabilities P2]	P2PS_prok	abilities,	v_opt] = pp	sr2 (0.5, 0	.020*randn	(201));					
The persecutor pure strateg	ies to be se	lected:									
The optimal probabilities of the optimal prob	0.4250 E the persec	0.4500 utor pure :	0.4800 strategies	0.5100 selection:	0.5300	0.5350	0.5400	0.5600	0.5800	0.6750	
۲۲۶_ probabilities = 0.0565 0.0030 0.1138	0.0274	0.1056	0.0998	0.0704	0.2346	0.0334	0.0812	0.1092	0.0145	0.0506	
The prey pure strategies to P2PS =	be selected	<u></u>									
The optimal probabilities or other and the optimal probabilities or other and the other other and the other other other and the other othe	0.0200 E the prey p	0.0350 Wre strate	0.0400 gies select	0.9650 ion:	0.9750	0.9800	0.9850	0066.0	0.9950	1.0000	
0.0477 0.1540 0.0536	0.0337	0.0396	0.1778	0.0095	0.0445	0.0838	0.1628	0.0232	0.0530	0.1169	
v_opt = 0.89417											
>> [P1PS P1PS_probabilities P2]	es P2PS_prot	abilities '	v_opt] = pp	sr2 (0.5, 0	.020*randn	;((102)					
The persecutor pure strateg pips =	ies to be se	lected:									
	0.4500 f the persec	0.4550 utor pure :	0.4650 strategies	0.4800 selection:	0.4950	0.5000	0.5600	0.5850	0.6000	0.6250	
PIPS_probabilities = 0.0058 0.0958 0.1964	0.0245	0.0658	0.0142	0.1363	0.1053	0.0232	0.0206	0.1328	0.1354	0.0440	
The prey pure strategies to P2PS =	be selected										
0 0.0050 0.0100 The optimal probabilities o	0.0150 f the prey p	0.0200 Nure strate	0.0250 gies select	0.0300 ion:	0.9700	0.9800	0.9850	0066.0	0.9950	1.0000	
rera_promanificies - 0.0282 0.0727 0.0987	0.0705	0.0110	0.097	0.1247	0.0645	0.0981	0.0762	0.0342	0.1329	0.0887	
v_opt = 0.892											
\$											
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Figure 46. Program module <code>ppsr2</code> application with  $\,\alpha=0.5\,$  by pretty high noise amplitude

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<pre>&gt;&gt; [P1PS P1PS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(0.5, 0.0040*randn(201));</pre>	
The persecutor pure strategies to be selected:	
P1PS = 0.4500 0.5400 0.5950 The optimal probabilities of the persecutor pure strategies selection: P1PS probabilities =	
0.6136 0.1287 0.2577	
The prey pure strategies to be selected: P2PS =	
u u.uusu 1.uuuu The optimal probabilities of the prey pure strategies selection: P2PS_probabilities = 0.1806 0.3154 0.5040	
$v_opt = 0.88565$	
<pre>&gt;&gt; [P1PS P1PS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(0.5, 0.0045*randn(201));</pre>	
The persecutor pure strategies to be selected:	
LIPS	
The optimal probabilities of the persecutor pure strategies selection: PIPS_probabilities = 0.1051 0.0727 0.0433	
The prey pure strategies to be selected:	
P2PS = 0 0.0050 0.9800 0.9950 1.0000	
The optimal probabilities of the prey pure strategies selection: P2P5_probabilities =	
0.1506 0.3191 0.0267 0.4370 0.0666	
$v_{opt} = 0.88695$	
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Figure 47. Optimal probabilities by  $\,\alpha=0.5\,$  for some decreased noise amplitude

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<pre>&gt;&gt; [P1PS P1PS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(0.5, 0.0050*randn(201));</pre>
The persecutor pure strategies to be selected:
P1PS = 0.4900 0.5000 0.5500 0.5500
The optimal probabilities of the persecutor pure strategies selection: PIPS probabilities =
$\overline{0.2662}$ 0.0488 0.1346 0.4221 0.1283
The prey pure strategies to be selected:
Terr 0 0.0150 0.9900 0.9950 1.0000 The optimal probabilities of the prev pure strategies selection:
P2PS_probabilities =
$v_{-}$ opt = 0.88624
<pre>&gt;&gt; [P1PS P1PS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(0.5, 0.0055*randn(201));</pre>
The verseritor mire stratewiss to be selected.
0.4100 0.4700 0.4950 0.5100 0.5150 0.5300 The optimal probabilities of the persecutor pure strategies selection:
PIPS_probabilities = 0.0210 0.2269 0.0931 0.2186 0.4347 0.0057
The prey pure strategies to be selected: P2P5 =
0 0.0050 0.0100 0.9900 0.9950 1.0000 The optimal probabilities of the prev pure stratedies selection:
P2PS_probabilities =
v_opt = 0.88659
A Start

Figure 48. Further increasing the noise amplitude, and the spectrums of the optimal strategies of both the persecutor and prey keep widen

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<pre>&gt;&gt; [PIPS PIPS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(0.75, 0.001*randn(201));</pre>	1
The persecutor pure strategies to be selected: PIPS =	
0.4800 0.5300 The optimal probabilities of the persecutor pure strategies selection: P1P2_probabilities = 0.5983 0.4017	
The prey pure strategies to be selected: P2PS =	
0 1 The optimal probabilities of the prey pure strategies selection: P2PS_probabilities = 0.5075 0.4925	
v_opt = 0.82979	
<pre>&gt;&gt; [PIPS PIPS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(1, 0.001*randn(201));</pre>	
The persecutor pure strategies to be selected: P1PS =	
P1FS_probabilities = 0.3564 0.6436	
The prey pure strategies to be selected: P2PS =	
0 1 The optimal probabilities of the prey pure strategies selection: P2PS_probabilities = 0.4993 0.5007	
v_opt = 0.77979	
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<pre>&gt;&gt; [PIPS PIPS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(2, 0.001*randn(201));</pre>
The persecutor pure strategies to be selected:
P1PS = 0.3750 0.6250
The optimal probabilities of the persecutor pure strategies selection: PIPS_probabilities = 0.5028 0.4972
The prey pure strategies to be selected:
P2PS = 0 1
The optimal probabilities of the prey pure strategies selection: P2PS_probabilities =
0.4991 0.5009
$v_{-}$ opt = 0.60802
<pre>&gt;&gt; [PlPS PlPS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(2, 0.003*randn(201));</pre>
The persecutor pure strategies to be selected:
PIPS = 0.4550 0.6450
The optimal probabilities of the persecutor pure strategies selection: P1PS_probabilities = 0.7585 0.2415
The prey pure strategies to be selected: P2PS =
0 1 The optimal probabilities of the prey pure strategies selection: P2PS_probabilities = 0.4990 0.5010
$v_{-}$ opt = 0.61141
A Start

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<pre>&gt;&gt; [P1PS P1PS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(2, 0.005*randn(201));</pre>	
The persecutor pure strategies to be selected:	
0.4300 0.5800 0.6950 0.7300	
The optimal probabilities of the persecutor pure strategies selection:	
0.6277 0.2449 0.1037 0.0237	
The prey pure strategies to be selected:	
P2PS =	
u u.uouu u.9900 1.0000 The optimal probabilities of the prey pure strategies selection:	
P2PS_probabilities = 0.3712 0.175 0.4888	
V_opt = 0.61398	
<pre>&gt;&gt; [P1PS P1PS_probabilities P2PS P2PS_probabilities v_opt] = ppsr2(2, 0.01*randn(201));</pre>	
The verseritor wire strateries to be selected.	
PIPS =	
0.2950 0.3650 0.4900 0.5100 0.6500	
The optimal probabilities of the persecutor pure strategies selection:	
$rr_{plubul}$	
The prey pure strategies to be selected:	
P2DS =	
0 0.0050 0.9850 0.9950 1.0000 The optimal probabilities of the prey pure stratedies selection:	
P2PS_probabilities =	
0.2188 0.2773 0.0022 0.0968 0.4048	
$v_{o}t = 0.61887$	
▲ Start	11

Figure 51. Further increasing the noise amplitude at  $\alpha = 2$ , and the spectrums of the optimal strategies of both the players keep widening Another new three examples of the module opr2 application are shown on the figures 52 — 54.

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>> [Payoff_1] = opr2(P, 8, 1, 0, 0, 0)					<u> </u>
The relative deviation of the averaged payoff of the Payoff_1 = 0.85174680994188 0.85174680994188 0.9307894123046 >> [Payoff_1] = opr2(P, 8, 1, 0, 0, 0)	first player is -0.00 5 0.89737297460298	66393 0.85174680994188	0.93078941230465	0.93078941230465	0.81528074254801
The relative deviation of the averaged payoff of the Payoff_1 =	first player is -0.05	7221			
<pre>&gt;&gt; [Payoff_1] = opr2(P, 8, 1, 0, 0, 0)</pre>	5 U.83006144557265	0.83006144557265	0.95400352364717	0.80948189591025	0.84940679849596
The relative deviation of the averaged payoff of the Payoff_1 =	first player is 0.019	356			
<pre>0.89580291239686 0.92160977803950 0.8755489839304 &gt;&gt; [Payoff_1] = opr2(P, 8, 1, 0, 0, 0)</pre>	5 0.98679776593654	0.87531707052803	0.92104883947434	0.94958223277458	0.94958223277458
The relative deviation of the averaged payoff of the Payoff_1 =	first player is 0.035	821			
0.94399587944094 0.90248443428280 0.9376781708940 >> [Payoff_1] = opr2(P, 8, 1, 0, 0, 0)	9 0.97571998951870	0.93440395849826	0.97571998951870	0.97571998951870	0.87965947930181
The relative deviation of the averaged payoff of the Payoff 1 =	first player is 0.003	7455			
0.93538721274712 0.94115280968018 0.8874459459141 >> [Payoff_1] = opr2(P, 8, 1, 0, 0, 0)	4 0.95664800456574	0.79294218922987	0.89419001531040	0.89780848421393	0.90697939042046
The relative deviation of the averaged payoff of the Payoff_1 =	first player is 0.098	741			
0.89347646571055 1.0000000000000 1.0000000000 >> [Payoff_1] = opr2(P, 8, 1, 0, 0, 0)	1.0000000000000000000000000000000000000	0.99416069967659	1.0000000000000000000000000000000000000	1.0000000000000000000000000000000000000	1.0000000000000
The relative deviation of the averaged payoff of the Pavoff 1 =	first player is -0.01	9166			
0.90955276694727 0.89329387401470 0.8842097547019	5 0.78141054312176	1.0000000000000000	0.90018171485924	0.78141054312176	0.88420975470195 💌







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>> [Payoff_1] = opr2(P, 10, 1, 0, 0, 1)	
Now the first player has selected the pure strategy x17	
Now the second player has selected the pure strategy y44	
Now the first player real payors 18 0. /2253	
Now the second player has selected the pure strategy #14	
Now the first player real payoff is 0.79354	
Now the first player has selected the pure strategy x11	
Now the second player has selected the pure strategy y49	
Now the first player real payoff is 0.79354	
Now the first player has selected the pure strategy x11	
Now the first player real paroff is 0.79354	
Now the first player has selected the pure strategy x11	
Now the second player has selected the pure strategy y6	
Now the first player real payoff is 1	
Now the first player has selected the pure strategy x23	
Now the second player has selected the pure strategy 944 Now the first night parties is 0.85375	
Now the first player has selected the pure strategy x11	
Now the second player has selected the pure strategy 744	
Now the first player real payoff is 0.75982	
Now the first player has selected the pure strategy x11	
Now the second player has selected the pure strategy y?	
Now the first player real payoff is 1	
Now the iscond player has selected the pure strategy v7	
Now the first player real payoff is 1	
Now the first player has selected the pure strategy x19	
Now the second player has selected the pure strategy y48	
At last, the first player real payoff is 0.93638	
The relative deviation of the averaged payoff of the first player is -U.U3361	
rayor Columns 1 through 8	
0.79252786153978 0.79353800957288 0.79353800957288 0.79353800957288 1.0000000000000 0.85375059281326 0.75982488499809 1.000000000000000	J
Columns 9 through 10	
1.00000000000 0.93637793179002	
>>	
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Figure 54. The module opr2 application with the 10 shots by some high noise level and the scheme of selecting the pure strategies

The executive form within MATLAB 7.0.1, gathered all the parameters and options of the antagonistic game explored persecution model, is viewed on the figure 55. This application, named eppsr2, works only with the mouseclicks by pulling the two scrolls to define the parameter  $\alpha$  and the roughness n(x, y) amplitude (root-mean-square deviation). The returned pure strategies in the optimal strategies of the persecutor and the prey are visualized on the surface (2)  $[0; 1] \times [0; 1]$ -square plot, helping to conceive them more physically (figures 56 - 65). Here, on the being presented form, a user does not define the roughness n(x, y) precisely by its sampled points, but holds or controls the level of that noise. This feature may occur useful, when a user just trains to possess the described simplest antagonistic game persecution model, for its further application from the MATLAB 7.0.1 Command Window line, inputting the real roughness n(x, y)sampled points into the program module ppsr2 and running it.



Figure 55. The executive form eppsr2 general view as it just has been loaded by the typing eppsr2 and entering it from the MATLAB 7.0.1 Command Window line



Figure 56. The single optimal pure strategy of the persecutor and the two prey pure strategies to be selected by the low α and the almost lowest noise level



Figure 57. The equilibrium situation by some noise level increment

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Figure 58. Appearance of the four pure strategies to be selected for each player by further noise level increasing



Figure 59. Disappearance of the one pure strategy to be selected for each player by some  $\, \alpha \,$  increment

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Figure 60. Again each player has the four pure strategies to be selected by further noise level increasing



Figure 61. Further increasing  $\,\alpha\,$  drives to the pure strategies to be selected reduction

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Figure 62. By the maximal  $\, \alpha \,$  there may be some stratification of the pure strategies to be selected



Figure 63. At the maximal noise level and not great  $\, lpha \,$  there appear more of the pure strategies to be selected

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Figure 64. At the maximal noise level and the lowest  $\alpha$  the number of the pure strategies to be selected becomes maximal for each player



Figure 65. Another example with the maximal noise level and the lowest  $\, \alpha \,$ 

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#### Conclusion

This paper generalizes the persecution model in the form of the antagonistic game with the kernel (2), that is where the kernel is the non-ideal exponential surface. For the generalization there has been updated the program module ppsr2, taking on the input  $\alpha$  and the roughness n(x, y). The module ppsr2 returns the exact analytic solution, if the game is either concave or convex (maybe, even concave-convex), and it returns the approximate solution for other cases, presenting the optimal strategies of the players (persecutor and prey). To practice those optimal probabilities the players should be involved into the program module opr2 procedure, accepting the optimal mixed strategy (or its spectrum) and returning the pure strategy number to be selected in the current game (current replay). The executive form eppsr2 helps to understand deeper the whole realization process of the antagonistic game persecution model, returning in the end the optimal strategies of the persecutor and the prey, and plotting them on the surface (2) graph.

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